Some mechanisms of fermion mass generation.

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Theoretical Physics Seminar Czech Technical University in Praga, 29-01-2019.

Based on: A. E. Cárcamo Hernández, EPJC **C76** (2016) no.9, 503.
A. E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, arxiv:hep-ph/1611.09797, JHEP **1702** (2017) 125.
A. E. Cárcamo Hernández, S. Kovalenko, R. Pasechnik, I. Schmidt, arxiv:hep-ph/1901.02764,

Overview

The Standard Model

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS force carriers spin = 0, 1, 2, ...

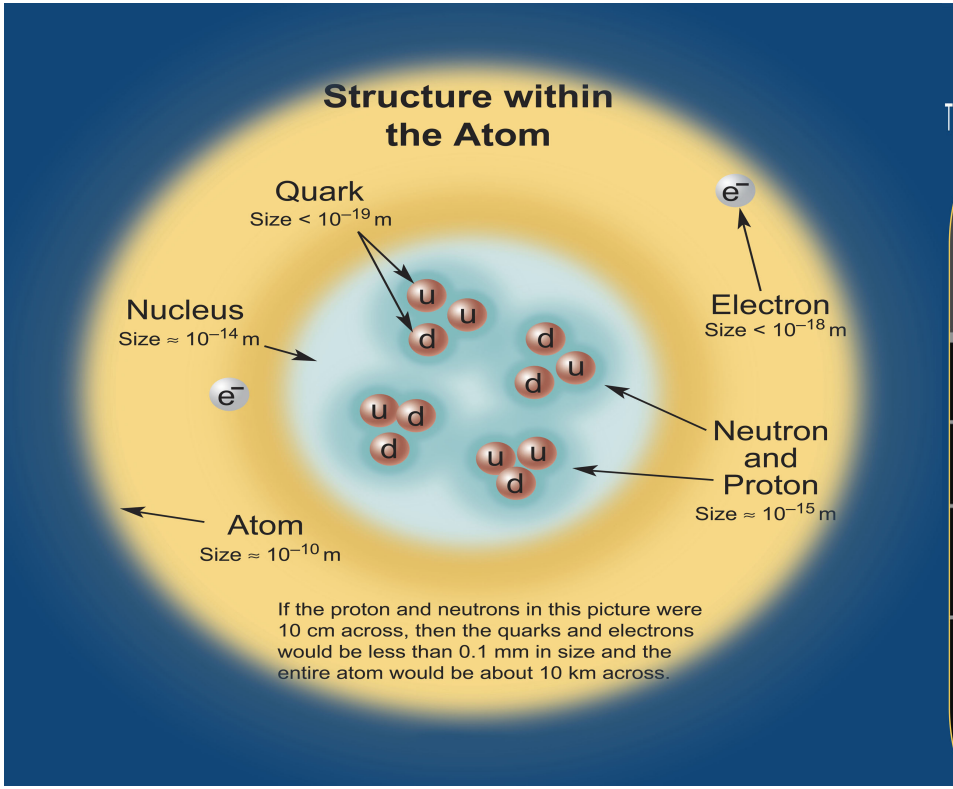
Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

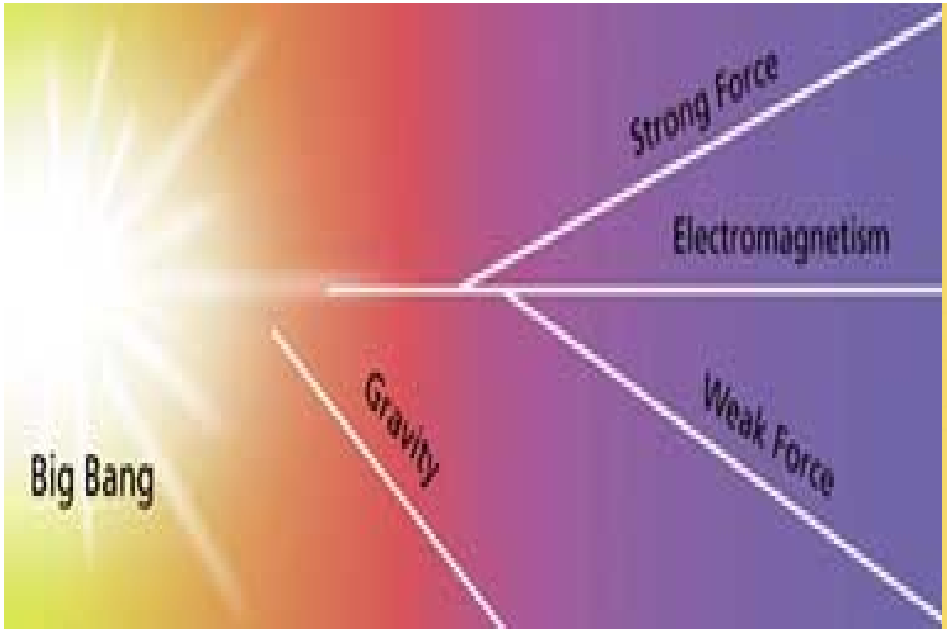
- Matter is made of fermions.
- Forces are mediated by bosons.
- Higgs boson breaks the electroweak symmetry and gives mass to fermions and weak gauge bosons.

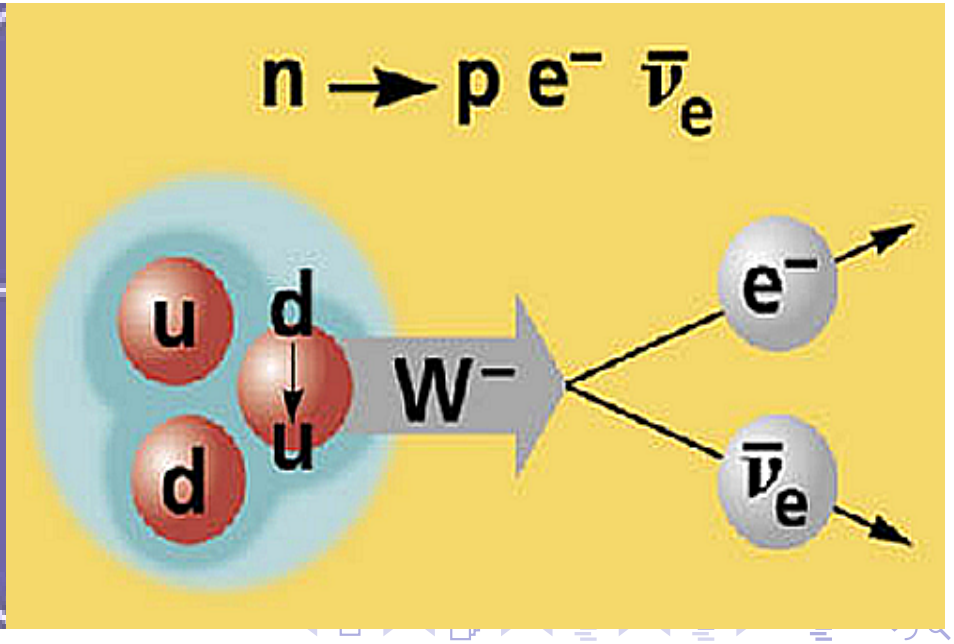


Properties of the Interactions

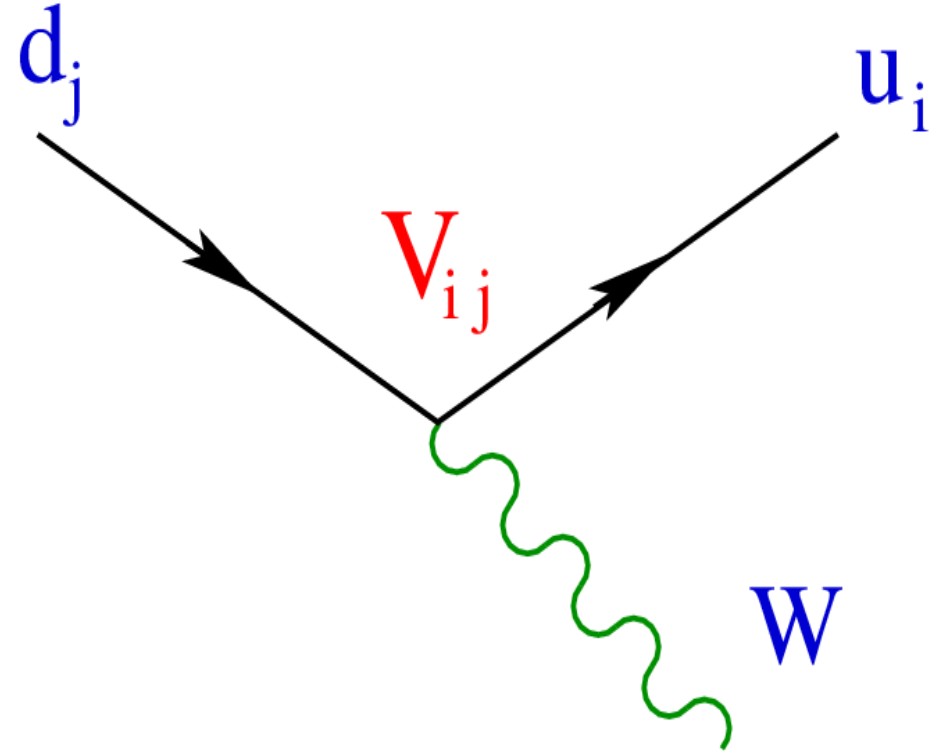
The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

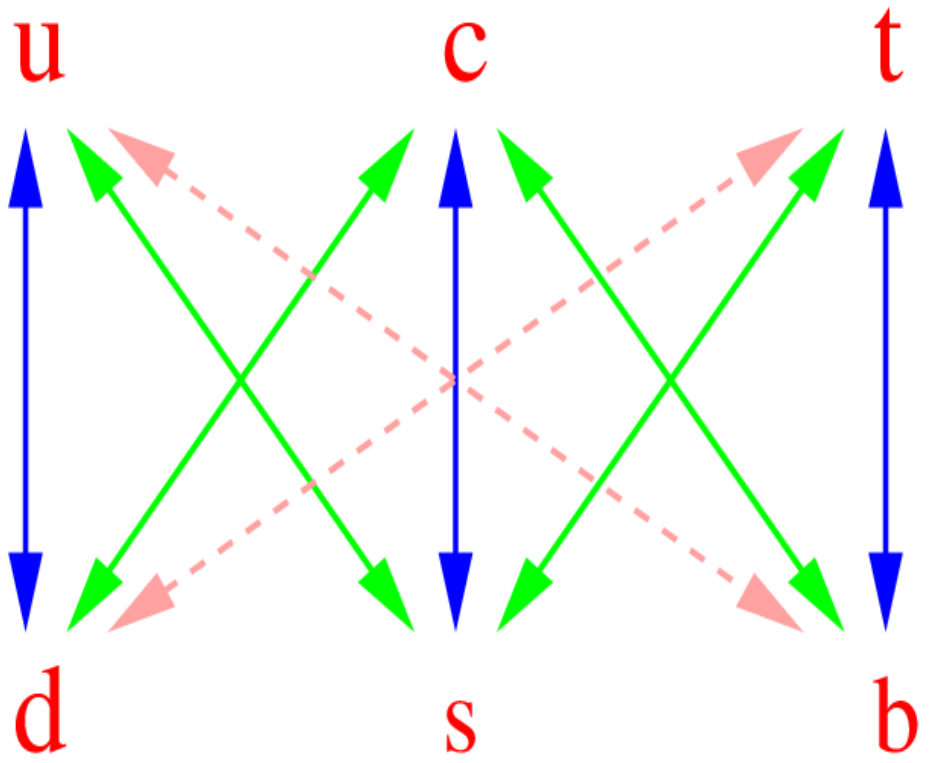
Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

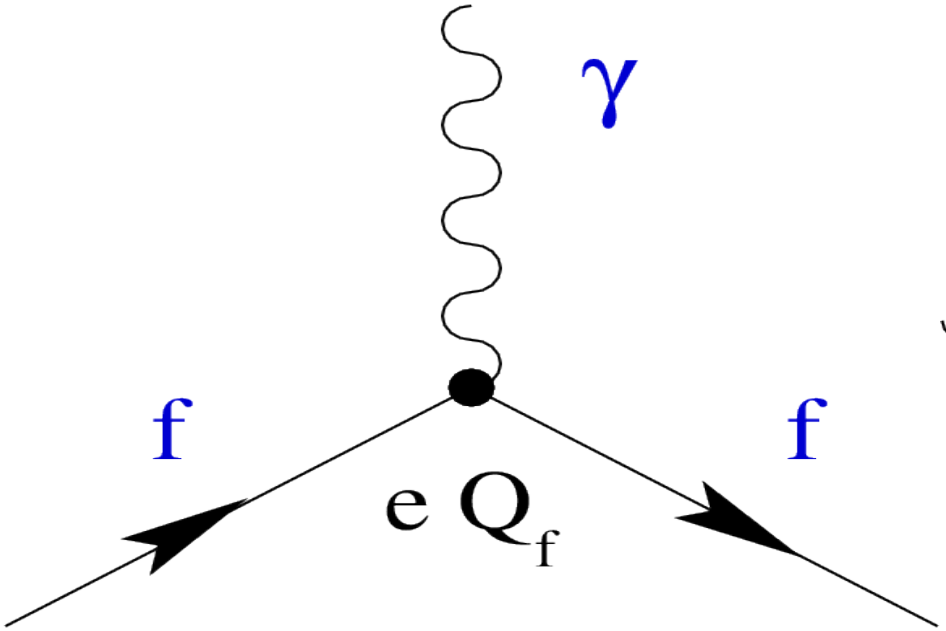


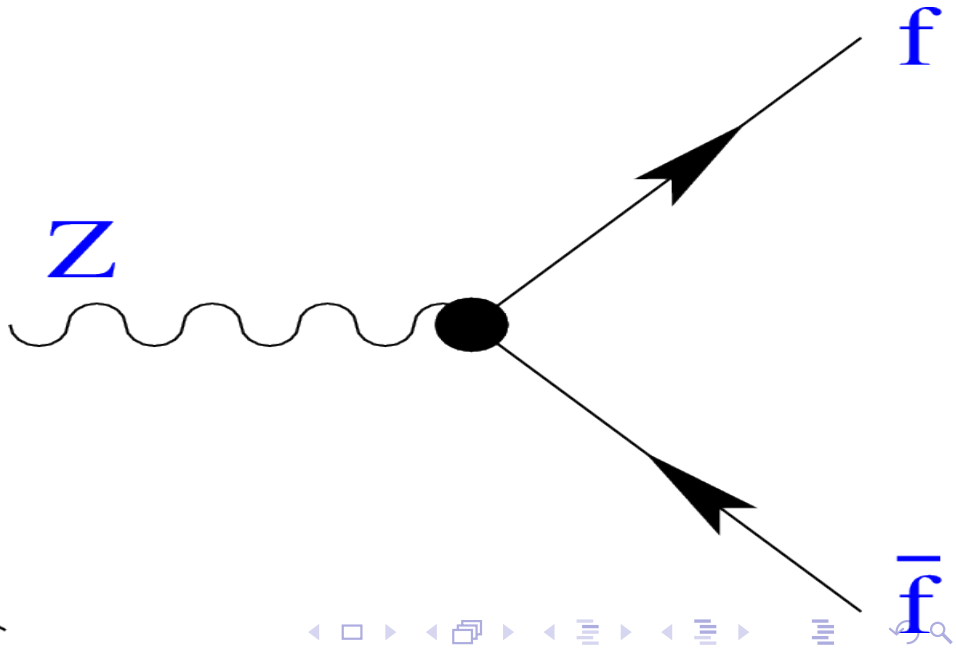


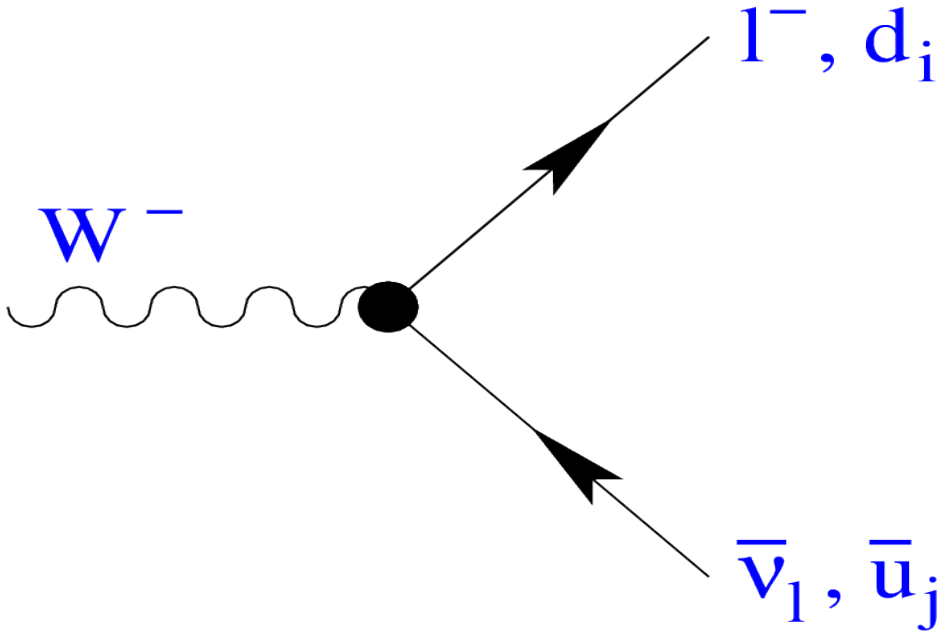
$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + i\bar{\Psi}\not{D}\psi \\
& + D_\mu\Phi^\dagger D^\mu\Phi - V(\Phi) \\
& + \bar{\Psi}_L\hat{Y}\Phi\Psi_R + h.c.
\end{aligned}
$$

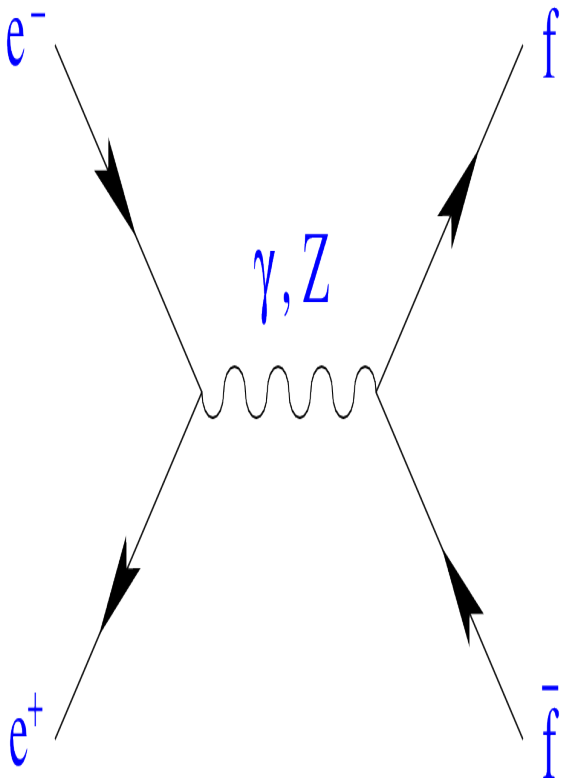

d_j
u_i
V_{ij}
W

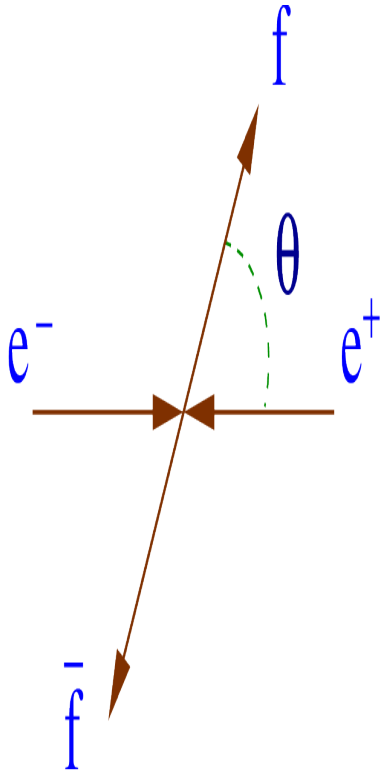

u
c
t
d
s
b


γ
f
f
$e\,Q_f$

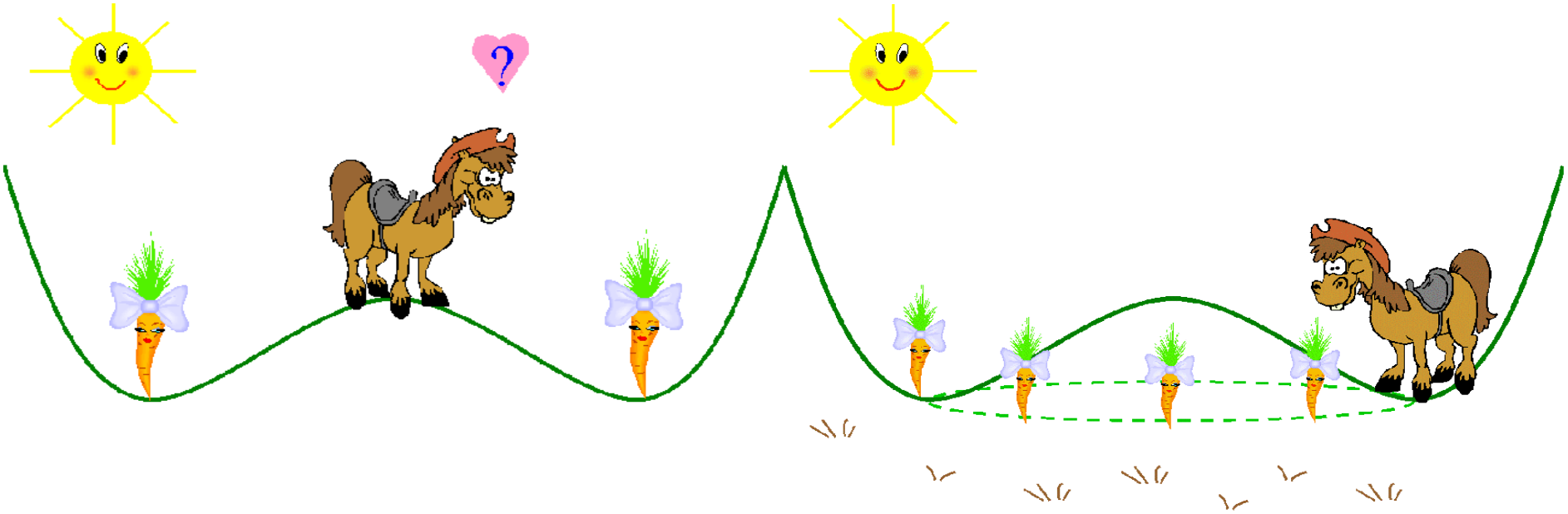

Z
f
$\bar{f}$

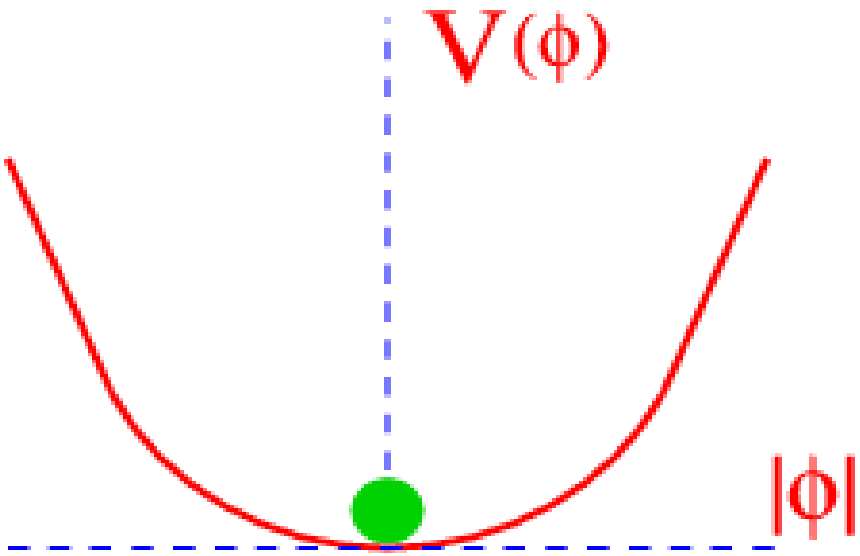

l^-, d_i
W^-
$\bar{\nu}_l, \bar{u}_j$

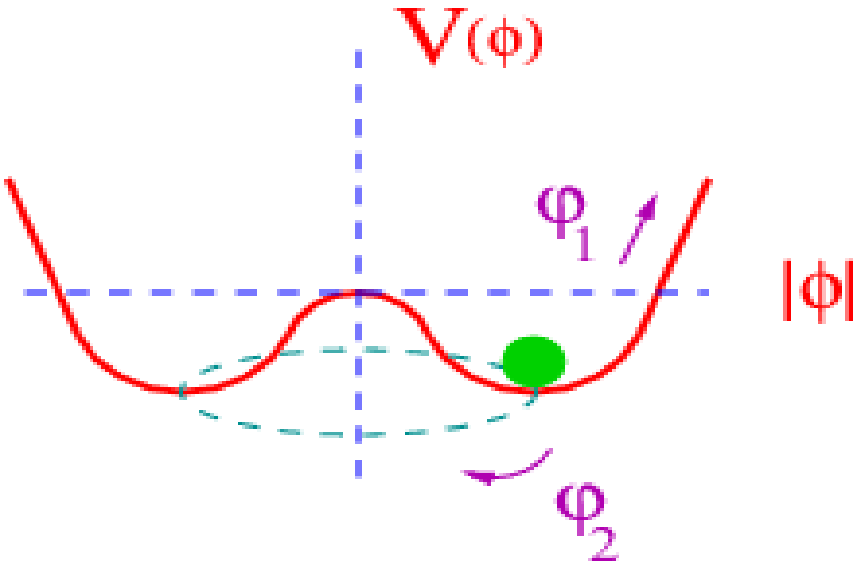

e^-
f
γ, Z
e^+
$\bar{f}$


f
θ
e^-
e^+
$\bar{f}$

Spontaneous Symmetry Breaking





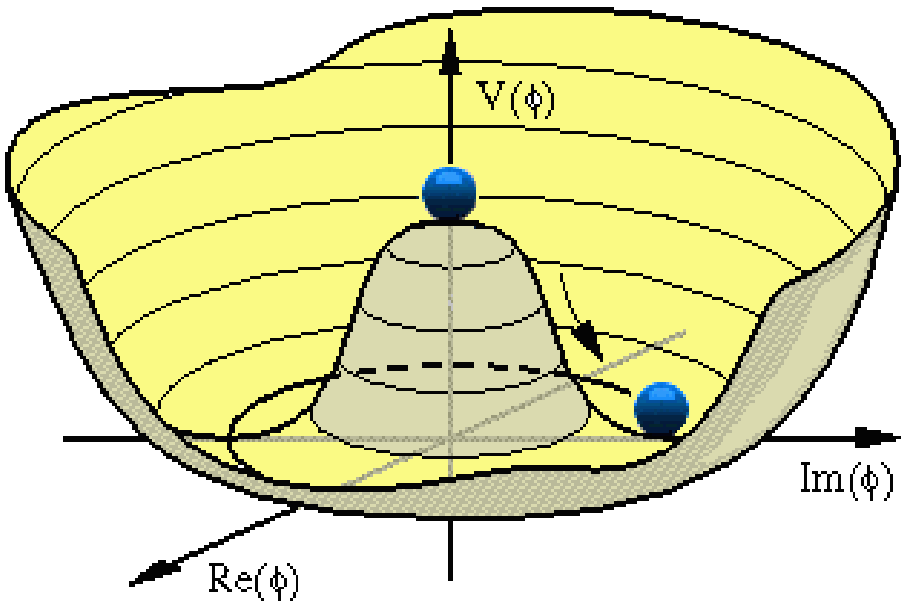


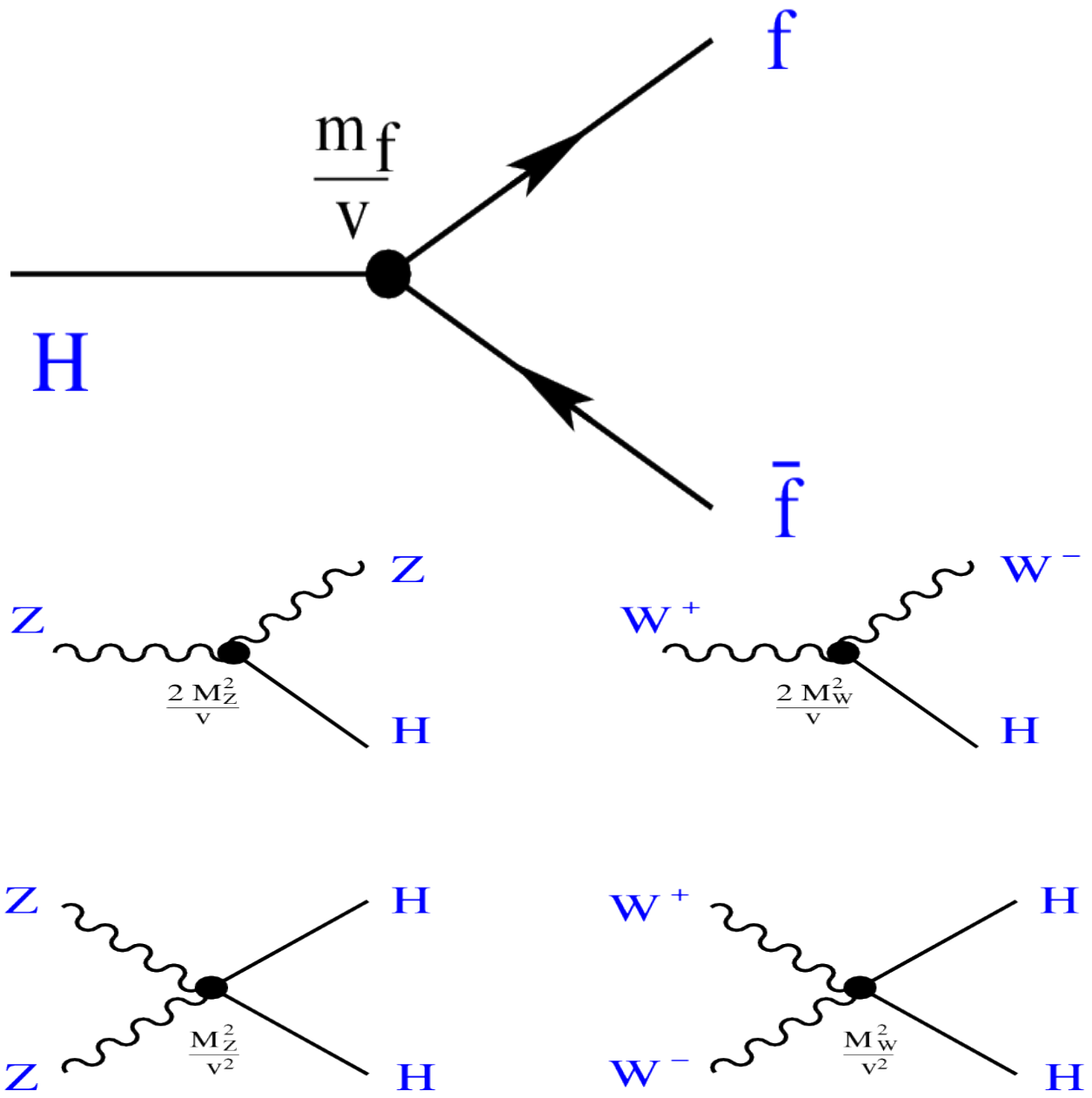
The Higgs Boson

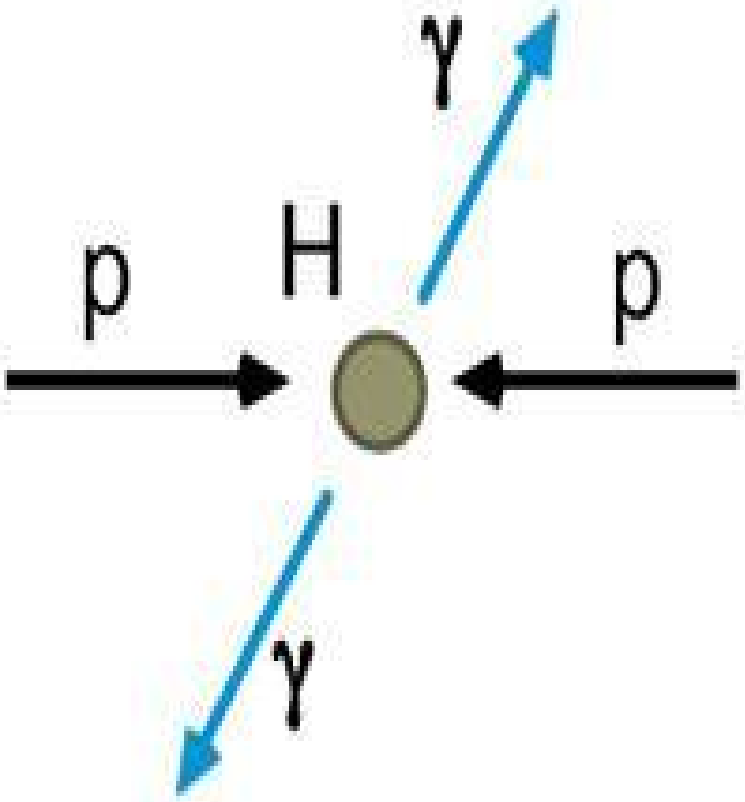
$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG^0\right) \end{pmatrix}, \qquad v = 246\,GeV, \tag{1}$$

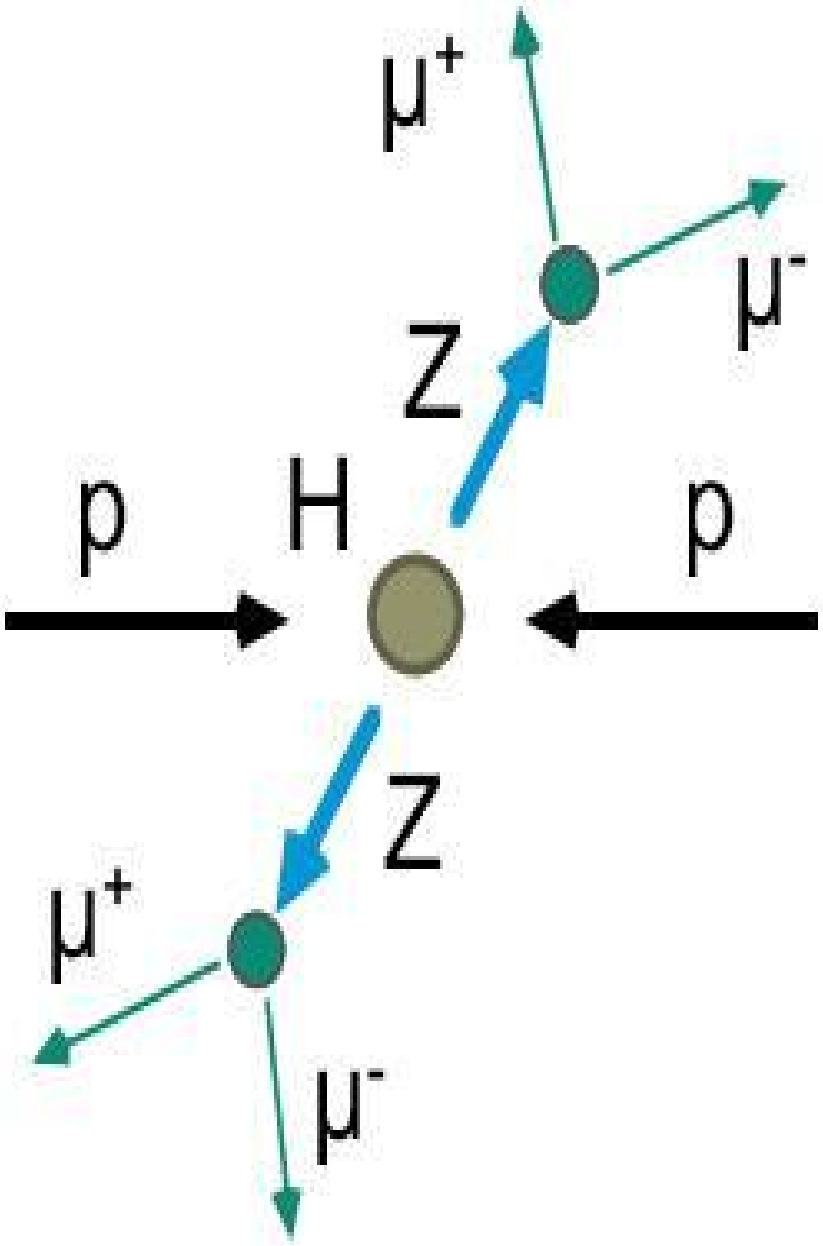
$$V = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4}\left(\Phi^\dagger \Phi\right)^2 \tag{2}$$

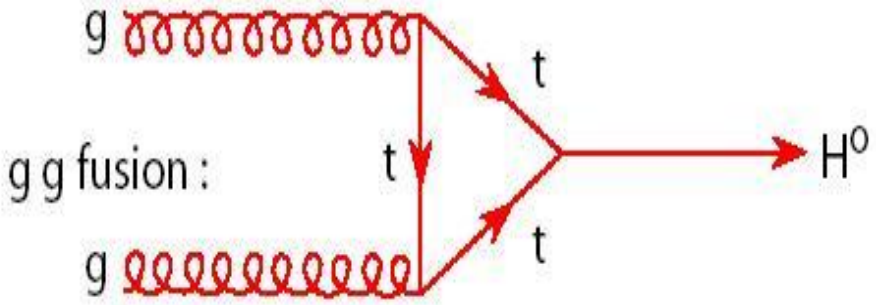
$$D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2} g W_\mu^a \tau^a \Phi - \frac{i}{2} g' B_\mu \Phi \tag{3}$$

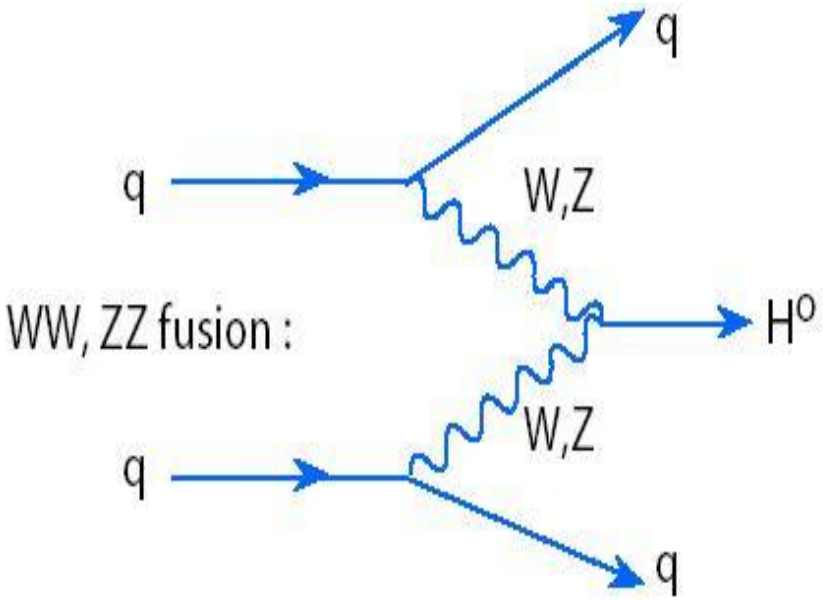


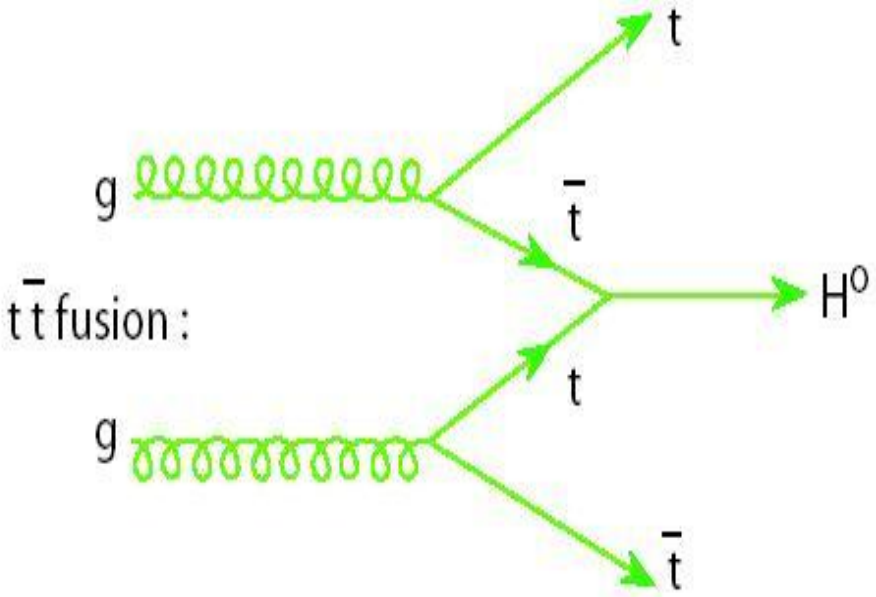

f
$\frac{m_f}{v}$
H
$\bar{f}$
Z
Z
$\frac{2\,M_Z^2}{v}$
H
W^+
W^-
$\frac{2\,M_W^2}{v}$
H
Z
H
$\frac{M_Z^2}{v^2}$
Z
H
W^+
H
$\frac{M_W^2}{v^2}$
W^-
H

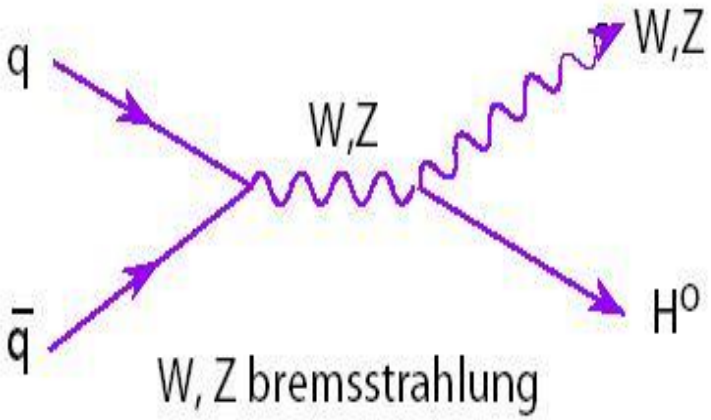

p
H
γ
p
γ

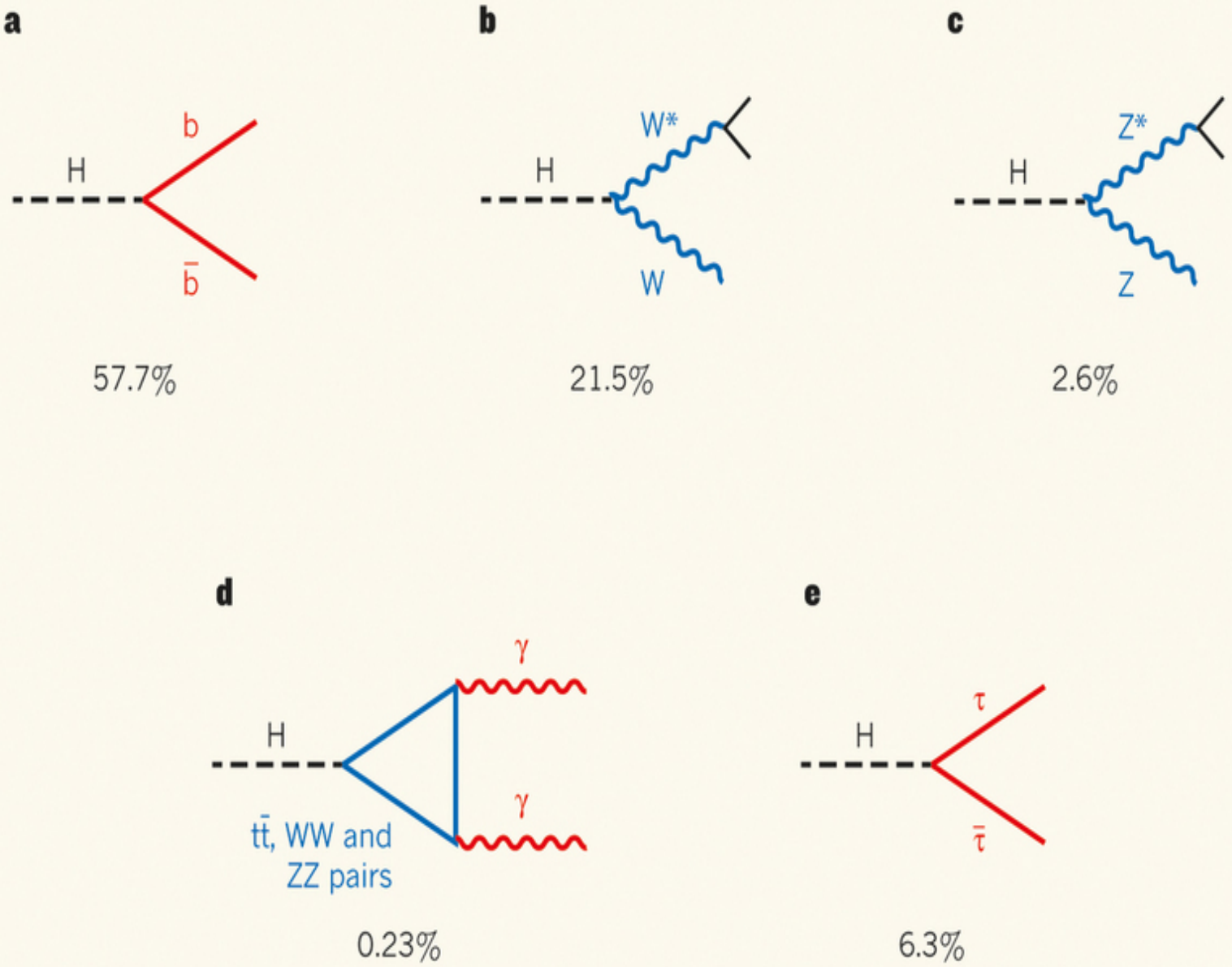

μ+
μ-
Z
p
H
p
Z
μ+
μ-

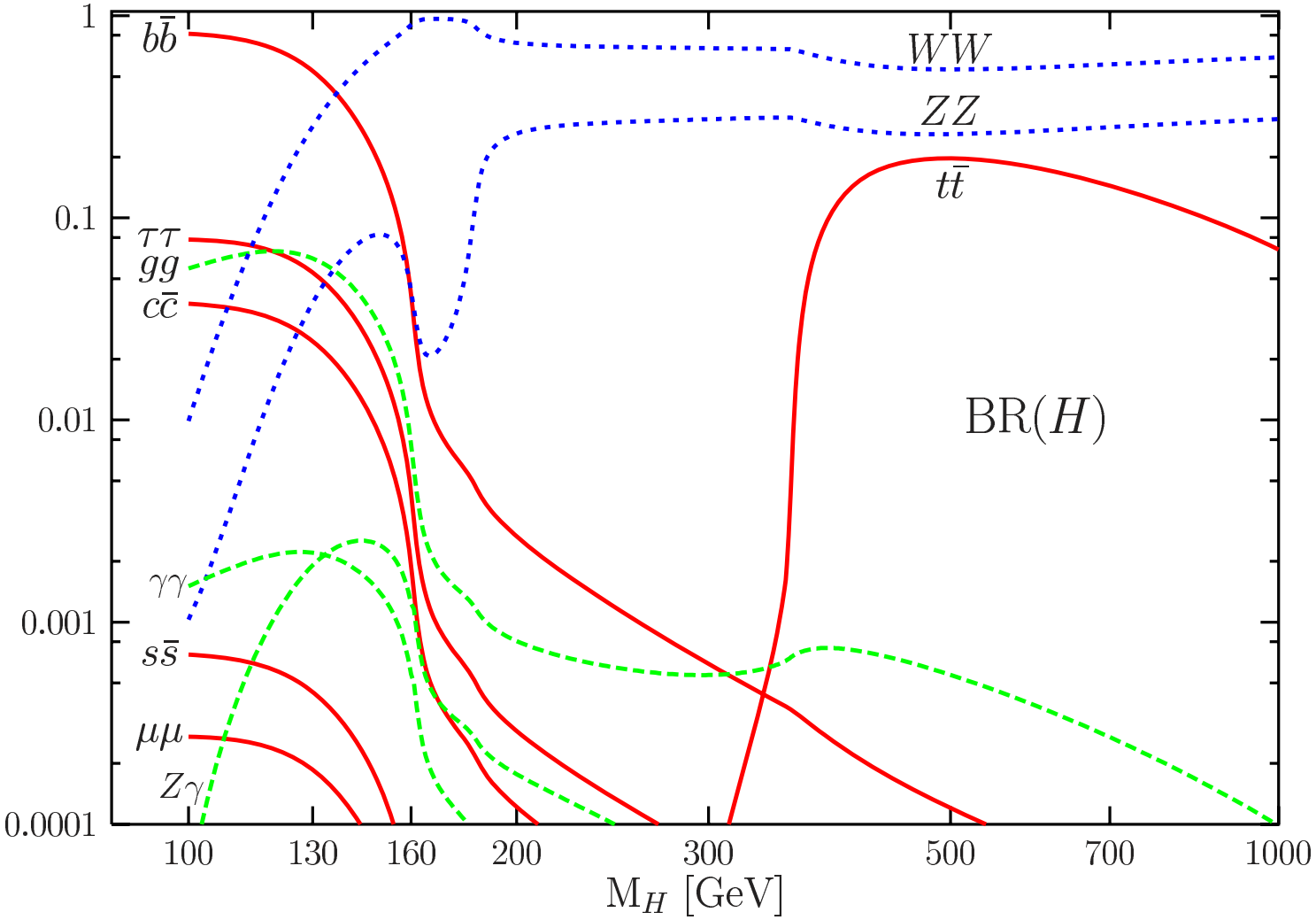

g
t
g g fusion :
t
H0
t
g

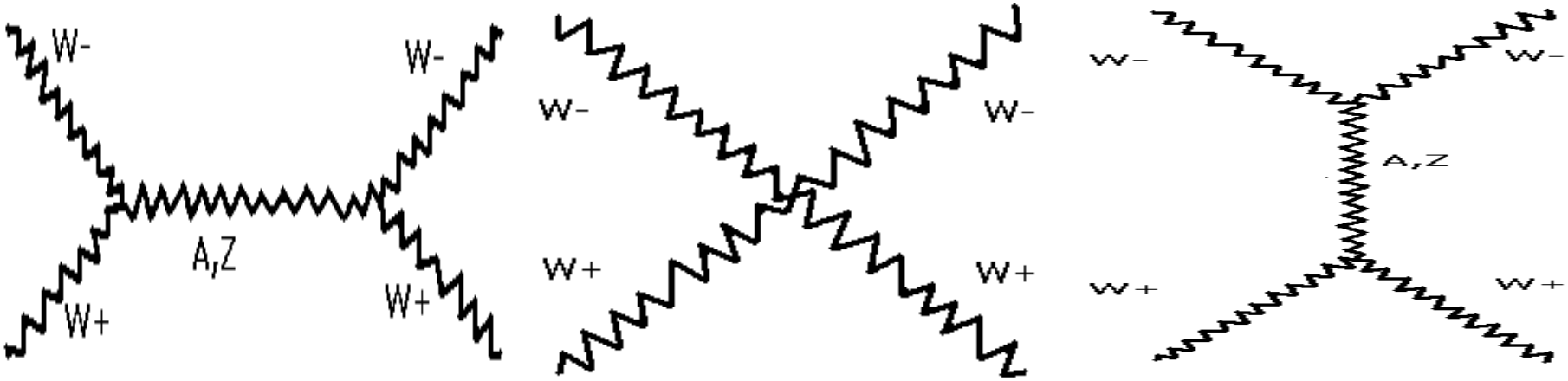

q
q
W,Z
WW, ZZ fusion :
H0
W,Z
q
q


t
g
t̄
t t̄ fusion :
H0
t
g
t̄

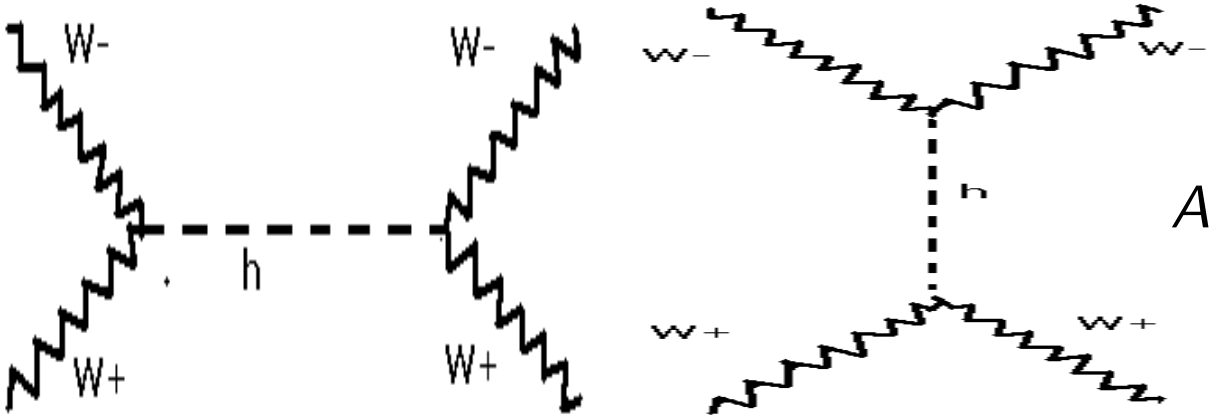

q
W,Z
W,Z
H0
q̄
W, Z bremsstrahlung


a
H
b
b̄
57.7%
b
W*
H
W
21.5%
c
Z*
H
Z
2.6%
d
γ
H
γ
tt̄, WW and
ZZ pairs
0.23%
e
τ
H
τ̄
6.3%


1
0.1
0.01
0.001
0.0001
$b\bar{b}$
$\tau\tau$
gg
$c\bar{c}$
$\gamma\gamma$
$s\bar{s}$
$\mu\mu$
$Z\gamma$
WW
ZZ
$t\bar{t}$
BR(H)
100
130
160
200
300
500
700
1000
M_H [GeV]



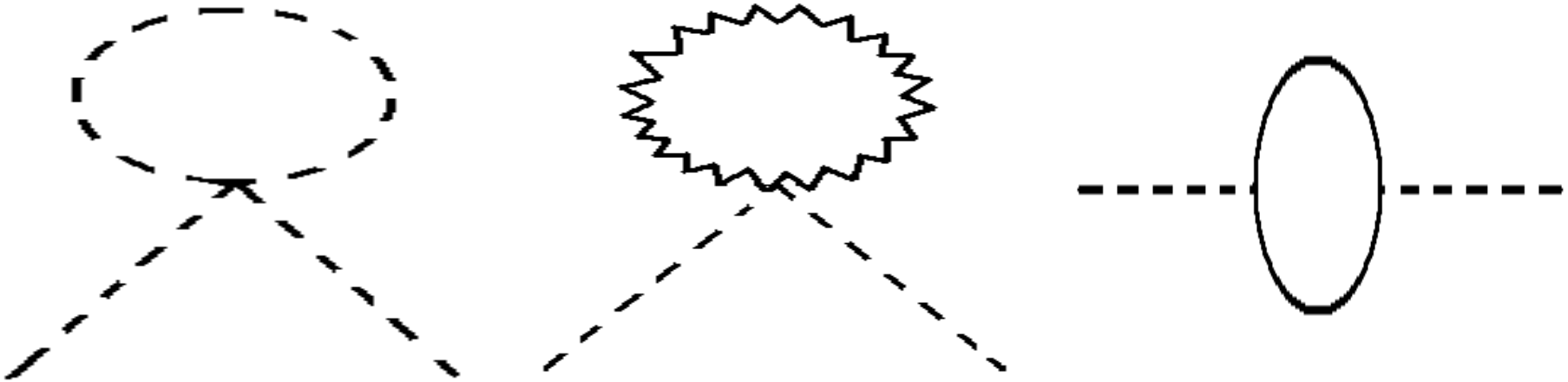
$$A_{Gauge} \simeq g^2 \left(\frac{E}{M_W} \right)^2$$



$$A_{Higgs} \simeq -g^2 \left(\frac{E}{M_W} \right)^2$$

$$A = A_{Gauge} + A_{Higgs} \simeq g^2 \left(\frac{M_H}{2M_W} \right)^2$$

The Higgs boson unitarize the WW scattering.



$$m_H^2 \sim \; m_0^2 - (115\,GeV)^2 \left(\frac{\Lambda}{400\,GeV}\right)^2$$

To have $m_H \approx 125$ GeV for $\Lambda \simeq 10^{19}$ GeV an extreme fine tunning of 34 decimals in the bare squared Higgs boson mass has to be performed. This is the hierarchy problem of the Standard Model.

The flavor problem

The origin of fermion masses and mixings is not explained by the SM.

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
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s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

$$\sqrt{\left|\Delta m_{13}^2\right|} \sim \lambda^{20} m_t, \quad \sqrt{\Delta m_{12}^2} \sim \lambda^{21} m_t,$$

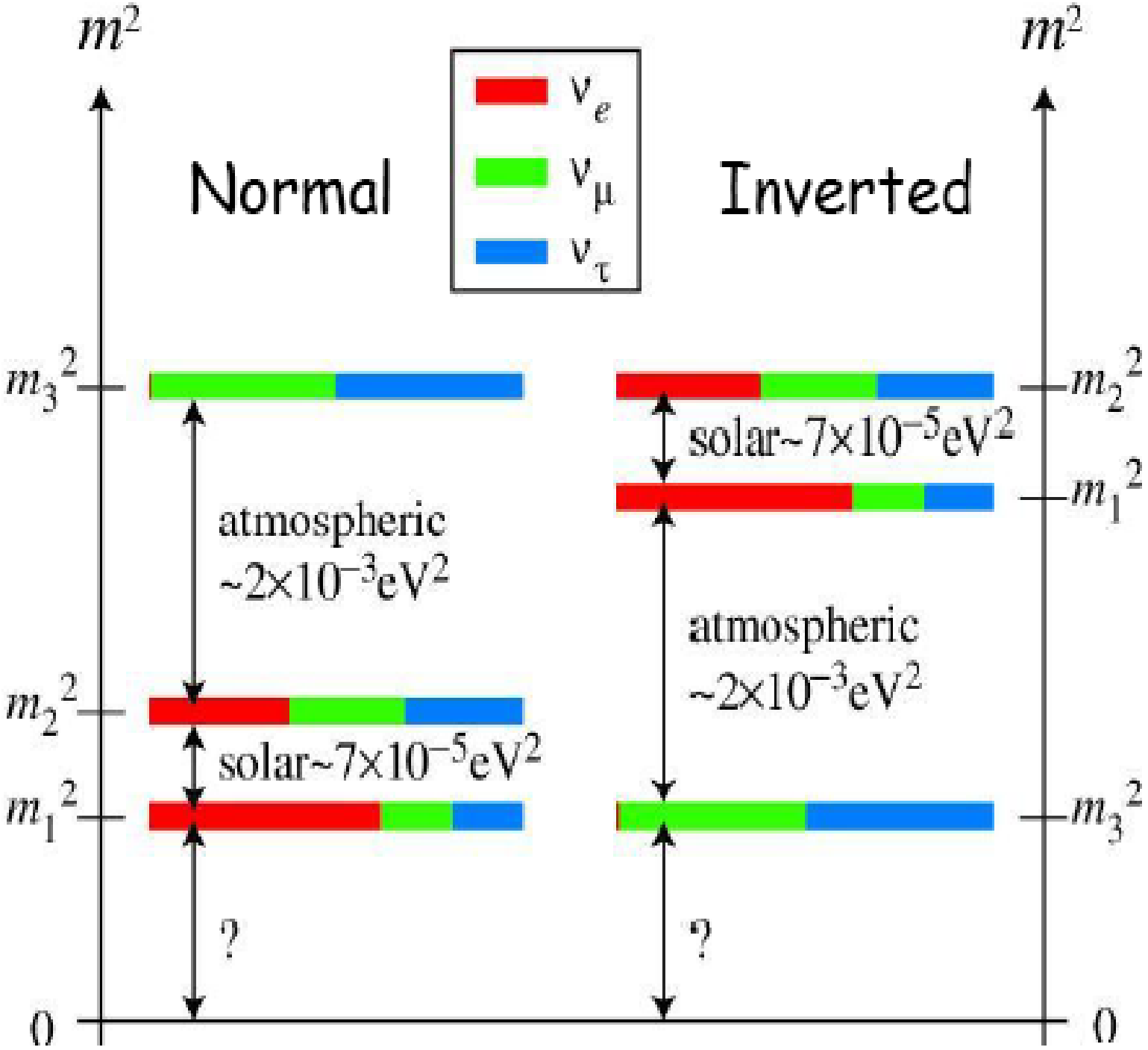
$$m_e \sim \lambda^9 m_t, \quad m_u \sim m_d \sim \lambda^8 m_t,$$

$$m_s \sim m_\mu \sim \lambda^5 m_t, \quad \lambda = 0.225,$$

$$m_c \sim \lambda^4 m_t, \quad m_b \sim m_\tau \sim \lambda^3 m_t,$$

$$\sin\theta_{12}^{(q)} \sim \lambda, \quad \sin\theta_{23}^{(q)} \sim \lambda^2, \quad \sin\theta_{13}^{(q)} \sim \lambda^4,$$

$$\sin\theta_{12}^{(l)} \sim \sqrt{\frac{1}{3}}, \quad \sin\theta_{23}^{(l)} \sim \sqrt{\frac{1}{2}}, \quad \sin\theta_{13}^{(l)} \sim \frac{\lambda}{\sqrt{2}}.$$


m^2
m^2
Normal
Inverted
ν_e
ν_μ
ν_τ
m_3^2
m_2^2
m_1^2
atmospheric
~2×10⁻³eV²
solar~7×10⁻⁵eV²
?
0
m_2^2
m_1^2
m_3^2
solar~7×10⁻⁵eV²
atmospheric
~2×10⁻³eV²
?
0

Some mechanisms to describe the SM charged fermion mass hierarchy are:

1. Spontaneously broken abelian symmetries as originally proposed by Froggatt and Nielsen in NPB, 1979.
2. Universal Seesaw mechanism as originally proposed by Davidson and Wali in PRL, 1987
3. Localization of the profiles of the fermionic zero modes in extradimensions as originally proposed by Dvali and Schifman in PLB, 2000.
4. Combining spontaneous breaking of discrete symmetries with radiative seesaw processes as in A.E. Cárcamo Hernández, EPJC, 2016 and C. Arbeláez, A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt, EPJC, 2017.
5. Sequential loop supression mechanism as originally proposed by A.E. Cárcamo Hernández, S. Kovalenko and I. Schmidt in JHEP, 2017.

Several mechanisms to generate light active neutrino masses are: Weinberg Operator, type I seesaw, type II seesaw, type III seesaw, double seesaw, linear seesaw, inverse seesaw, radiative seesaw at one, two, three or four loop level.

Froggatt-Nielsen mechanism

The Froggatt-Nielsen mechanism has the following features:

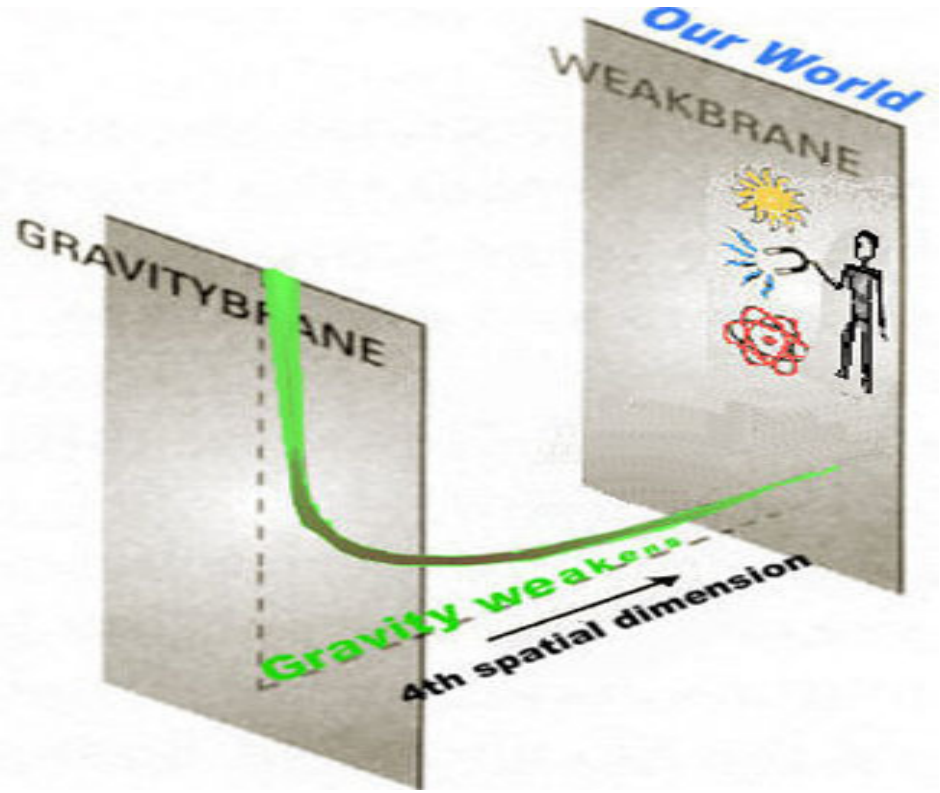
- Introduce new gauge singlet scalar, i.e., σ called the flavon, and a global $U(1)_{FN}$ symmetry.
- The $U(1)_{FN}$ charges of the SM fermions (excepting for the top Yukawa term), the Higgs and Flavon fields are such that renormalizable Yukawa terms are forbidden.
- The $U(1)_{FN}$ charge assignments of fermionic and scalar fields generate the following Effective operator:

$$a_{ij}\overline{f}_{iL}Hf_{jR}\left(\frac{\sigma}{\Lambda}\right)^{n_{ij}} \rightarrow a_{ij}\left(\frac{v_\sigma}{\Lambda}\right)^{n_{ij}}\overline{f}_{iL}Hf_{jR} \tag{4}$$

- The Yukawa hierarchy arises from the $U(1)_{FN}$ charge assignment:

$$n_{ij} = -\frac{1}{q_\varphi}\left(q_{\overline{f}_{iL}} + q_{f_{jR}} + q_H\right) \tag{5}$$

Warped Extradimensional Model



- SM fields are located at the TeV brane (Visible Universe) and gravity propagates along the extradimension.
- Space time is deformed as a exponential factor as $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$.
- Condensing fourth quarks generation breaks EW symmetry at the TeV brane and gives masses to quarks and weak gauge bosons.

The S_3 is the smallest non-abelian group having a doublet and two singlet irreducible representations. The S_3 group has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010):

$$\begin{aligned} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} &= (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} \\ &+ \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{2}}, \end{aligned} \tag{6}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_{\mathbf{2}}, \qquad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \tag{7}$$

A toy model: Generating $m_b \neq 0$ at one loop level with $m_d = m_s = 0$.

To get massless d, s and b quarks at tree level, we forbidd the operators

$$\overline{q}_{iL}\phi d_{jR}, \qquad i,j = 1,2,3\,, \tag{8}$$

To this end, we consider the following S_3 assignments:

$$q_{iL} \sim \mathbf{1}, \qquad d_{iR} \sim \mathbf{1}', \qquad \phi \sim \mathbf{1} \tag{9}$$

We assume S_3 softly broken and we add gauge singlet scalars η_k $(k = 1,2)$ and vector like down type quarks B_k $(k = 1,2)$ grouped in S_3 doblets as follows:

$$\eta = (\eta_1, \eta_2) \sim \mathbf{2}, \qquad B_{L,R} \sim \mathbf{2} \tag{10}$$

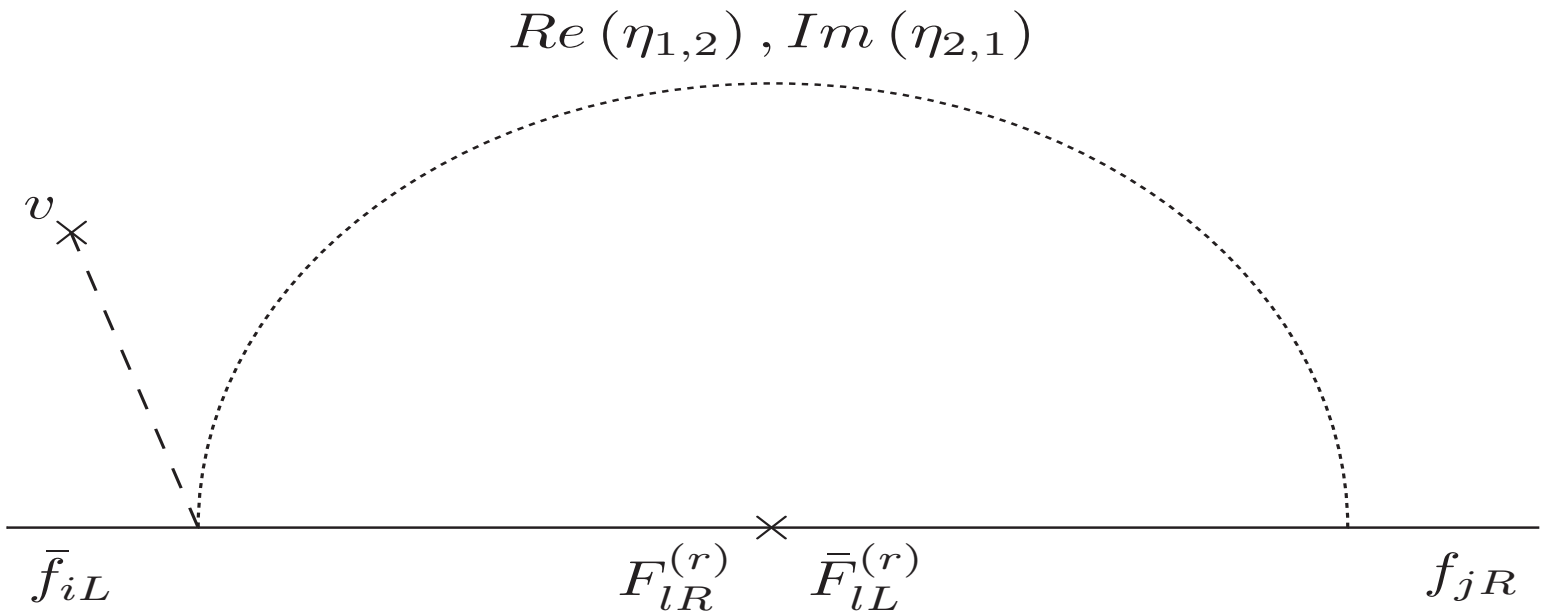
Thus, we are left with the operators:

$$\frac{y_i}{\Lambda}\overline{q}_{iL}\phi\left(B_R\eta\right)_{\mathbf{1}}, \qquad x_j\left(\overline{B}_L\eta\right)_{\mathbf{1}'}d_{jR}, \qquad i,j=1,2,3, \tag{11}$$

which imply:

$$\left(M_D\right)_{ij} \approx \frac{y_i x_j}{\left(16\pi^2\right)^2} f_2 \frac{v}{M}\frac{\mu_{12}^3}{\Lambda^3}\mu_{12}, \tag{12}$$

where μ_{12} is a soft breaking mass parameter in $\mu_{12}^2\eta_1\eta_2$. Thus $m_b \neq 0$ at one loop level and $m_d = m_s = 0$.



Combining radiative mechanisms with spontaneously broken symmetries.

The S_3 symmetry is softly broken whereas the Z_8 discrete group is broken.

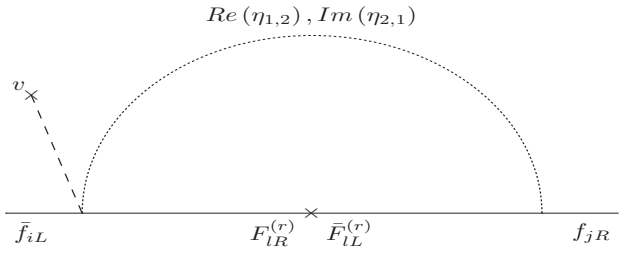
$$\phi \sim (\mathbf{1},1)\,, \quad \eta = (\eta_1, \eta_2) \sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad \chi \sim (\mathbf{1}, -i)\,,$$
$$v_\chi = \lambda\Lambda, \qquad \lambda = 0.225. \tag{13}$$

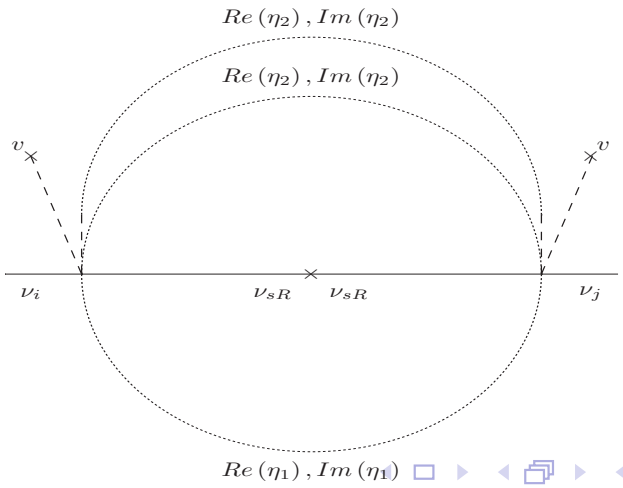
$$\begin{aligned}
q_{jL} &\sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad u_{kR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-k)}{2}}\right), \quad u_{3R} \sim (\mathbf{1}, 1)\,, \\
d_{jR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \quad l_{jL} \sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad l_{jR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \\
T_L^{(k)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad T_R^{(k)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad k = 1, 2, \\
B_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad B_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad j = 1, 2, 3, \\
E_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad E_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \\
\nu_{kR} &\sim \left(\mathbf{1}', e^{-\frac{\pi i}{4}}\right), \quad k = 1, 2.
\end{aligned} \tag{14}$$

I use the S_3 discrete group since it is the smallest non-Abelian group.

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}}^{(U)} &= \sum_{j=1}^{3}\sum_{r=1}^{2} y_{jr}^{(u)}\overline{q}_{jL}\widetilde{\phi}\left(T_R^{(r)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{r=1}^{2}\sum_{s=1}^{2} x_{rs}^{(u)}\left(\overline{T}_L^{(r)}\eta\right)_{\mathbf{1}'} u_{sR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{j=1}^{3} y_{j3}^{(u)}\overline{q}_{jL}\widetilde{\phi}u_{3R}\frac{\chi^{3-j}}{\Lambda^{3-j}} + \sum_{r=1}^{2} y_r^{(T)}\left(\overline{T}_L^{(r)}T_R^{(r)}\right)_{\mathbf{1}}\chi + h.c
\end{aligned}
$$

$$
-\mathcal{L}_{\mathrm{Y}}^{(\nu)} = \sum_{j=1}^{3}\sum_{s=1}^{2} y_{js}^{(\nu)}\overline{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{\left[\eta^*\left(\eta\eta^*\right)_{\mathbf{2}}\right]_{\mathbf{1}'}\chi^{3-j}}{\Lambda^{6-j}} + \sum_{s=1}^{2} y_s\overline{\nu}_{sR}\nu_{sR}^{C}\chi + h.c.
$$





Cárcamo Hernández-Kovalenko-Schmidt mechanism

In the CKS mechanism the SM fermion mass hierarchy is explained by a sequential loop suppression, so that the masses are generated according to:

$$t\text{-quark} \rightarrow \textit{tree-level mass} \text{ from } \overline{q}_{jL}\widetilde{\phi}u_{3R}, \tag{15}$$

$$b, c, \tau, \mu \rightarrow \textit{1-loop mass;} \text{ tree-level suppressed by a } \textit{symmetry}. \tag{16}$$

$$s, u, d, e \rightarrow \textit{2-loop mass;} \text{ tree-level \& 1-loop suppressed by a } \textit{symmetry}. \tag{17}$$

$$\nu_i \rightarrow \textit{4-loop mass;} \text{ tree-level \& lower loops suppressed by a } \textit{symmetry}. \tag{18}$$

The $S_3 \times Z_2$ particle assignments of the model are:

	ϕ	σ	η
S_3	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
Z_2	1	1	-1

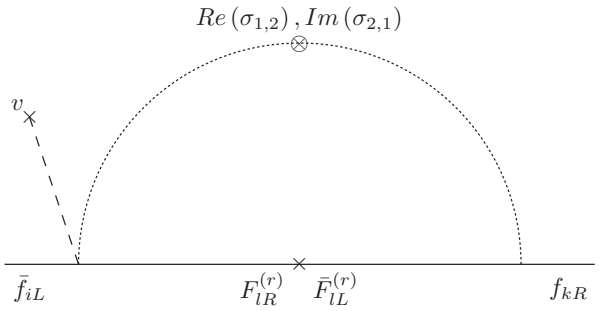
	q_{iL}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	l_{iL}	l_{1R}	l_{2R}	l_{3R}
S_3	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}'$
Z_2	1	-1	1	1	-1	-1	1	1	-1	1	1

	ν_{sR}	T_L	T_R	$\widetilde{T}_L$	$\widetilde{T}_R$	B_L	B_R	$\widetilde{B}_L^{(s)}$	$\widetilde{B}_R^{(s)}$	$E_L^{(s)}$	$E_R^{(s)}$	$\widetilde{E}_L$	$\widetilde{E}_R$
S_3	$\mathbf{1}'$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$
Z_2	-1	1	1	1	-1	1	1	1	-1	1	1	1	-1

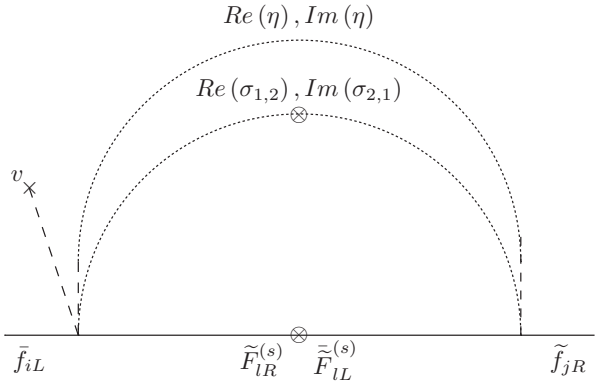
φ is the SM Higgs doublet.

The scalar fields σ and η and all exotic fermions are $SU(2)_L$ singlets.

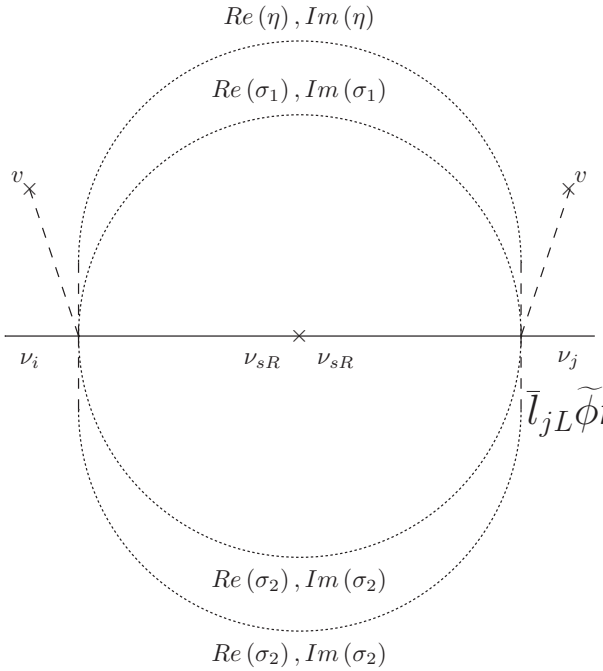
The $S_3 \times Z_2$ discrete group is assumed to be softly broken.



$$\overline{f}_{jL}H\left(F_R^{(r)}\sigma\right)_{\mathbf{1}}\frac{1}{\Lambda}\qquad\left(\overline{F}_L^{(r)}\sigma\right)_{\mathbf{1'}}f_{kR}$$



$$\overline{f}_{jL}H\left(\widetilde{F}_R^{(r)}\sigma\right)_{\mathbf{1}}\frac{\eta}{\Lambda^2}\qquad\left(\overline{\widetilde{F}}_L^{(r)}\sigma\right)_{\mathbf{1'}}\widetilde{f}_{kR}\frac{\eta}{\Lambda}$$



$$\bar{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{[\sigma(\sigma\sigma)_{\mathbf{2}}]_{\mathbf{1'}}\eta}{\Lambda^4}$$

The mass matrices $M_{U,D}$ of up and down quarks, $M_{l,\nu}$, of charged leptons and light active neutrinos

$$M_U = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} & \kappa_{13}^{(u)} \\ \widetilde{\varepsilon}_{12}^{(u)} & \varepsilon_{22}^{(u)} & \kappa_{23}^{(u)} \\ \widetilde{\varepsilon}_{13}^{(u)} & \varepsilon_{32}^{(u)} & \kappa_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(d)} & \widetilde{\varepsilon}_{12}^{(d)} & \varepsilon_{13}^{(d)} \\ \widetilde{\varepsilon}_{21}^{(d)} & \widetilde{\varepsilon}_{22}^{(d)} & \varepsilon_{23}^{(d)} \\ \widetilde{\varepsilon}_{31}^{(d)} & \widetilde{\varepsilon}_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

$$M_l = \begin{pmatrix} \widetilde{\varepsilon}_{11}^{(l)} & \varepsilon_{12}^{(l)} & \varepsilon_{13}^{(l)} \\ \widetilde{\varepsilon}_{21}^{(l)} & \varepsilon_{22}^{(l)} & \varepsilon_{23}^{(l)} \\ \widetilde{\varepsilon}_{31}^{(l)} & \varepsilon_{32}^{(l)} & \varepsilon_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_\nu = \begin{pmatrix} \varepsilon_{11}^{(\nu)} & \varepsilon_{12}^{(\nu)} & \varepsilon_{13}^{(\nu)} \\ \varepsilon_{12}^{(\nu)} & \varepsilon_{22}^{(\nu)} & \varepsilon_{23}^{(\nu)} \\ \varepsilon_{13}^{(\nu)} & \varepsilon_{23}^{(\nu)} & \varepsilon_{33}^{(\nu)} \end{pmatrix} \frac{v^2}{\sqrt{2}\,\Lambda},$$

their entries are generated at different loop-levels:

$$\kappa_{j3}^{(u)} \rightarrow \text{tree-level} \tag{19}$$

$$\varepsilon_{j2}^{(u)}, \varepsilon_{j3}^{(d)}, \varepsilon_{j2}^{(l)}, \varepsilon_{j3}^{(l)} \rightarrow \text{1-loop-level} \tag{20}$$

$$\widetilde{\varepsilon}_{j1}^{(u)}, \widetilde{\varepsilon}_{j1}^{(d)}, \widetilde{\varepsilon}_{j2}^{(d)}, \widetilde{\varepsilon}_{j1}^{(l)} \rightarrow \text{2-loop-level} \tag{21}$$

$$\varepsilon_{jk}^{(\nu)} \rightarrow \text{4-loop-level}, \tag{22}$$

where $j, k = 1, 2, 3$.

$$m_b \sim \frac{y_b^2}{16\pi^2} f_1 \frac{v}{\Lambda} \frac{\mu_{12}}{M} \mu_{12}, \tag{23}$$

$$m_s \sim \frac{y_s^2}{(16\pi^2)^2} f_2 \frac{v}{M} \frac{\mu_{12}^3}{\Lambda^3} \mu_{12}, \tag{24}$$

Assuming $y_b^2 f_1 \sim y_s^2 f_2 \sim 1$ and $\mu_{12} \sim M$, we find a rough estimate

$$\Lambda \sim 10v \sim 2.5\text{TeV} \tag{25}$$

for the correct order of magnitude of m_b and m_s.

Model Phenomenology.

$$M_\nu = \frac{\mu_\eta^2 \mu_\sigma^6 v}{(16\pi^2)^4 \Lambda^8} \begin{pmatrix} \beta_1^2+\gamma_1^2 & \beta_1\beta_2+\gamma_1\gamma_2 & \beta_1\beta_3+\gamma_1\gamma_3 \\ \beta_1\beta_2+\gamma_1\gamma_2 & \beta_2^2+\gamma_2^2 & \beta_2\beta_3+\gamma_2\gamma_3 \\ \beta_1\beta_3+\gamma_1\gamma_3 & \beta_2\beta_3+\gamma_2\gamma_3 & \beta_3^2+\gamma_3^2 \end{pmatrix},$$

$$\beta_s = y_{s1}^{(\nu)} \frac{v}{m_1} f_1^{(\nu)}, \quad \gamma_s = y_{s2}^{(\nu)} \frac{v}{m_2} f_2^{(\nu)}, \quad s = 1, 2. \tag{26}$$

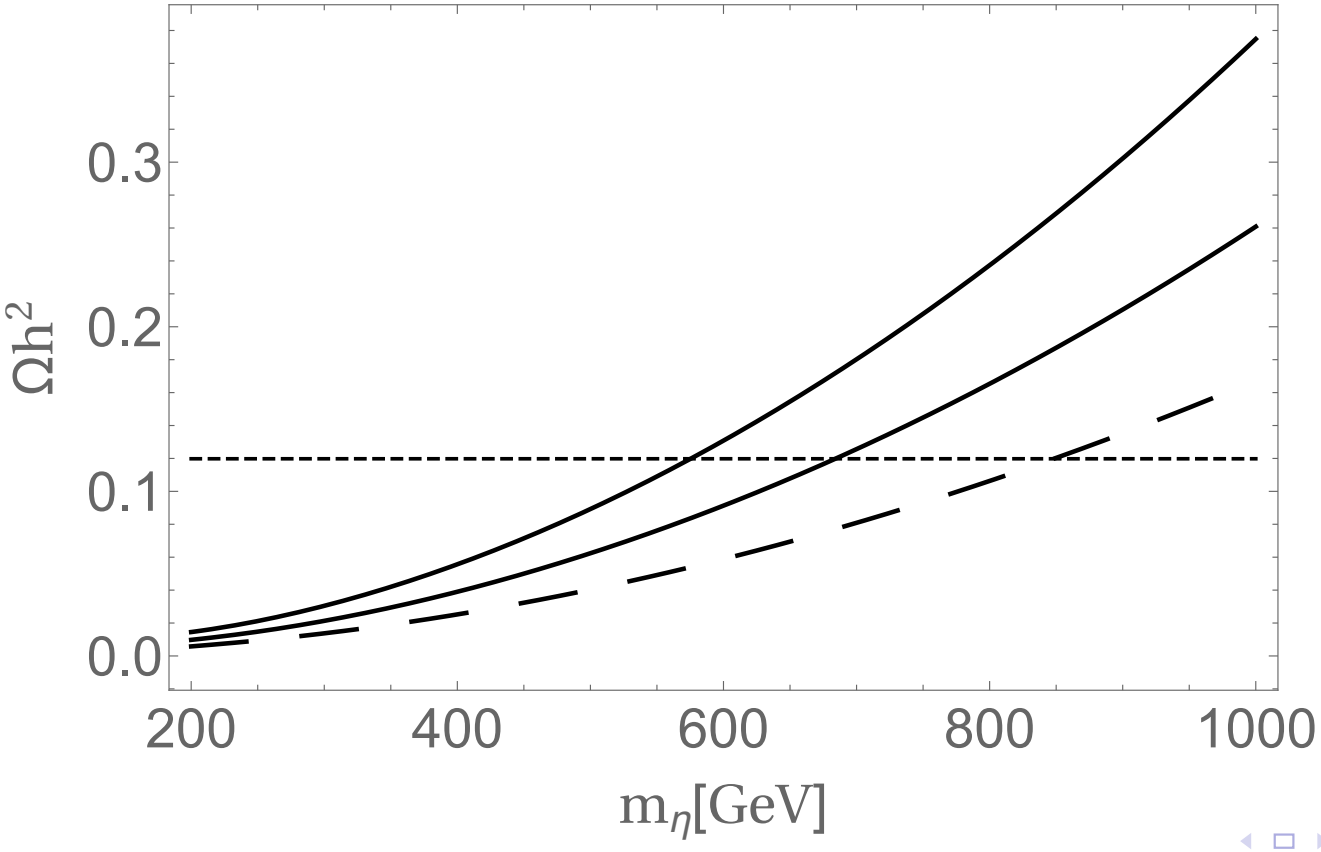
$$m_\nu \sim \frac{\left(y^{(\nu)}\right)^2}{(16\pi^2)^4} f^{(\nu)} \frac{v}{m_s} \frac{\mu_\eta^2 \mu_\sigma^6}{\Lambda^8} v. \tag{27}$$

Assuming $\left(y^{(\nu)}\right)^2 \cdot f^{(\nu)} \sim 1$, $\mu_\eta \sim \mu_\sigma \sim m_s \sim \alpha \cdot \Lambda$ and taking $\Lambda = 2.5$TeV from the quark sector (25) we find for $\alpha \sim 1$ the light neutrino mass scale $m_\nu \sim 1$eV, which is too heavy. Assuming, for instance, $\alpha = 0.3$ we arrive at the correct neutrino mass scale $m_\nu \sim 50$ meV. We expect a typical mass scale for all the non-SM particles – the η-DM candidate, in particular, – to be $m_{\text{non-SM}} \sim m_\eta \sim \alpha \cdot \Lambda \sim 750$ GeV.

The only possible decay modes of η are

$$\eta \rightarrow \sigma_{1,2}\widetilde{T}_{2L,1L}u_{1R},\ \sigma_{1,2}\widetilde{T}_{1R,2R}u_{iL},\ \sigma_{1,2}\widetilde{B}^{(s)}_{2L,1L}d_{kR},\ \sigma_{1,2}\widetilde{B}^{(s)}_{1R,2R}d_{iL},$$
$$\sigma_{1,2}\widetilde{E}_{2L,1L}l_{1R},\ \sigma_{1,2}\widetilde{E}_{1R,2R}e_{iL},\ \sigma_1 2\sigma_2\nu_{iL}\nu_{sR} \tag{28}$$

with $s, k = 1, 2$ and $i = 1, 2, 3$.
We assume that our DM candidate η annihilates mainly into WW, ZZ, $t\bar{t}$, $b\bar{b}$ and hh. We take $\lambda_{h^2\eta^2} = 1, 1.2, 1.5$ (from top to bottom, respectively).



An extended IDM with sequentially loop-generated fermion mass hierarchies.

$$\begin{aligned}\mathcal{G} &= SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times Z_2 \times Z_4 \\ &\xrightarrow{v_{\sigma_1}, v_{\rho_5}} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \\ &\xrightarrow{v_\eta} SU(3)_C \times U(1)_{em} \times Z_4, \end{aligned} \tag{29}$$

Field	ϕ_1	ϕ_2	σ_1	σ_2	σ_3	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	η	φ_1^+	φ_2^+
SU_{3c}	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU_{2L}	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U_{1Y}	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0	0	0	0	0	0
U_{1X}	1	2	-1	-1	-2	0	0	1	0	0	1	3	3
Z_2	1	1	1	1	-1	1	1	1	-1	-1	-1	1	1
Z_4	1	-1	1	-1	-1	i	-1	i	-1	1	-1	-1	1

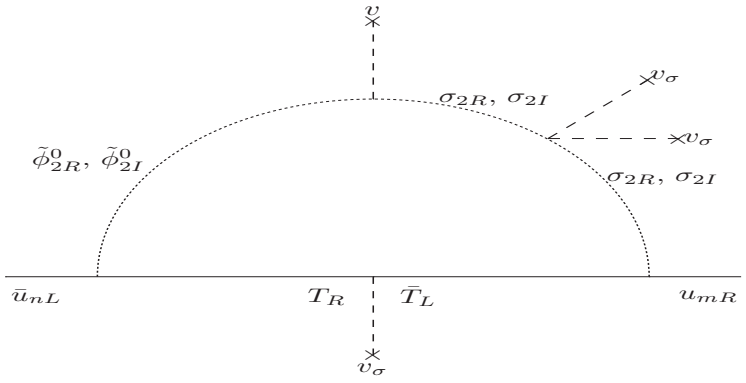
Table: Scalars assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2 \times Z_4$ symmetry.

Field	q_{1L}	q_{2L}	q_{3L}	u_{1R}	u_{2R}	u_{3R}	d_{1R}	d_{2R}	d_{3R}	T_L	T_R	$\widetilde{T}_{1L}$	$\widetilde{T}_{1R}$	$\widetilde{T}_{2L}$	$\widetilde{T}_{2R}$	B_{1L}	B_{1R}	B_{2L}	B_{2R}
SU_{3c}	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
SU_{2L}	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U_{1Y}	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
U_{1X}	0	0	1	2	2	2	-1	-1	-1	1	2	2	2	2	2	0	-1	0	-1
Z_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Z_4	-1	-1	1	-1	-1	1	1	1	1	1	1	$-i$	$-i$	-1	-1	-1	-1	-1	-1

Table: Quark assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2 \times Z_4$ symmetry.

Field	l_{1L}	l_{2L}	l_{3L}	l_{1R}	l_{2R}	l_{3R}	E_{1L}	E_{1R}	E_{2L}	E_{2R}	E_{3L}	E_{3R}	ν_{1R}	ν_{2R}	ν_{3R}	Ω_{1R}	Ω_{2R}	Ψ_R
SU_{3c}	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU_{2L}	**2**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U_{1Y}	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
U_{1X}	0	-3	0	-3	-6	-3	-3	-2	-6	-5	-3	-2	2	-1	2	-1	1	0
Z_2	1	-1	1	1	-1	1	1	1	-1	-1	1	1	1	-1	1	-1	-1	1
Z_4	-1	-1	-1	-1	-1	-1	$-i$	$-i$	1	1	1	1	1	1	1	-1	1	1

Table: Lepton assignments under the $SU_{3c} \times SU_{2L} \times U_{1Y} \times U_{1X} \times Z_2 \times Z_4$ symmetry.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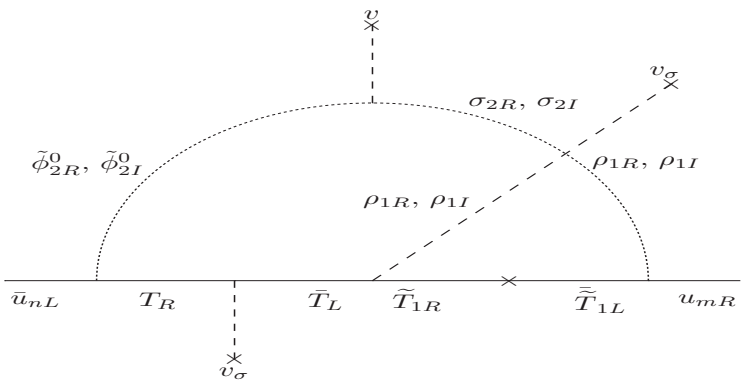
v_σ
$\sigma_{2R},\ \sigma_{2I}$
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
$\bar{u}_{nL}$
T_R
$\bar{T}_L$
u_{mR}
v_σ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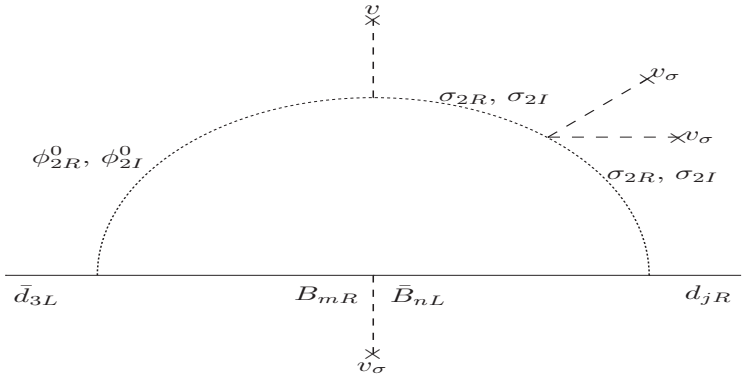
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\rho_{1R},\ \rho_{1I}$
$\tilde{\phi}^0_{2R},\ \tilde{\phi}^0_{2I}$
$\rho_{1R},\ \rho_{1I}$
$\bar{u}_{nL}$
T_R
$\bar{T}_L$
$\widetilde{T}_{1R}$
$\bar{\widetilde{T}}_{1L}$
u_{mR}
v_σ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v_σ
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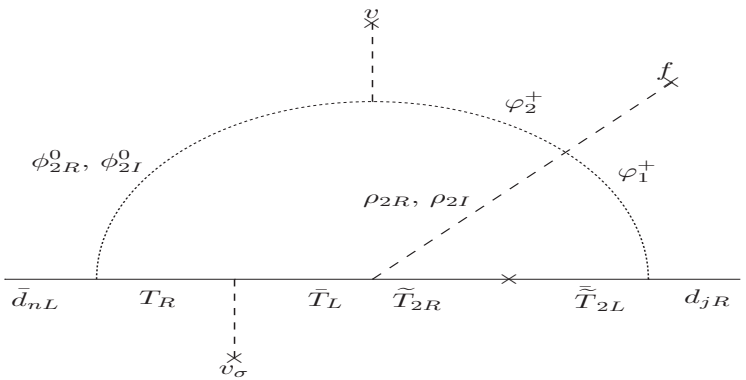
v_σ
$\sigma_{2R},\ \sigma_{2I}$
$\phi^0_{2R},\ \phi^0_{2I}$
$\bar{d}_{3L}$
B_{mR}
$\bar{B}_{nL}$
d_{jR}
v_σ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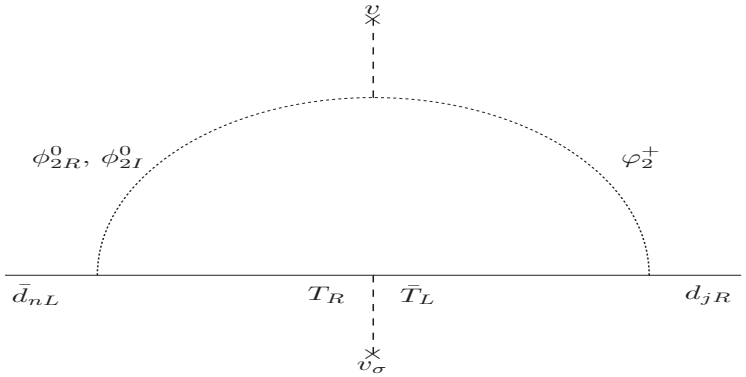
f
φ_2^+
φ_1^+
$\phi^0_{2R},\ \phi^0_{2I}$
$\rho_{2R},\ \rho_{2I}$
$\bar{d}_{nL}$
T_R
$\bar{T}_L$
$\widetilde{T}_{2R}$
$\bar{\widetilde{T}}_{2L}$
d_{jR}
v_σ


v
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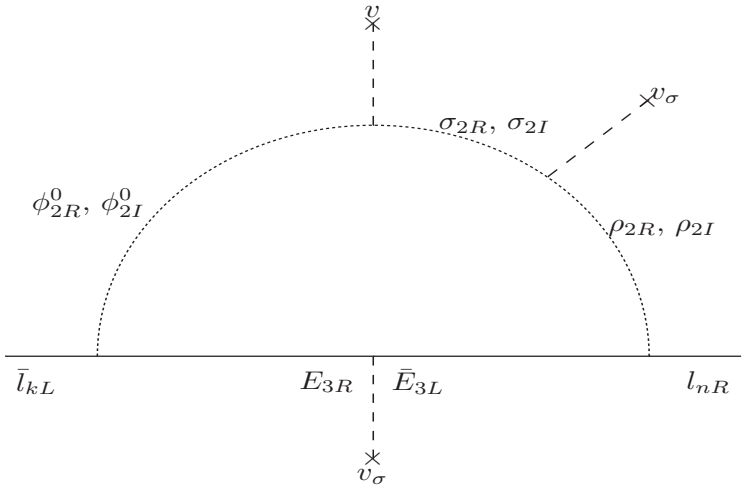
φ_2^+
$\bar{d}_{nL}$
T_R
$\bar{T}_L$
d_{jR}
v_σ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$\phi^0_{2R},\ \phi^0_{2I}$
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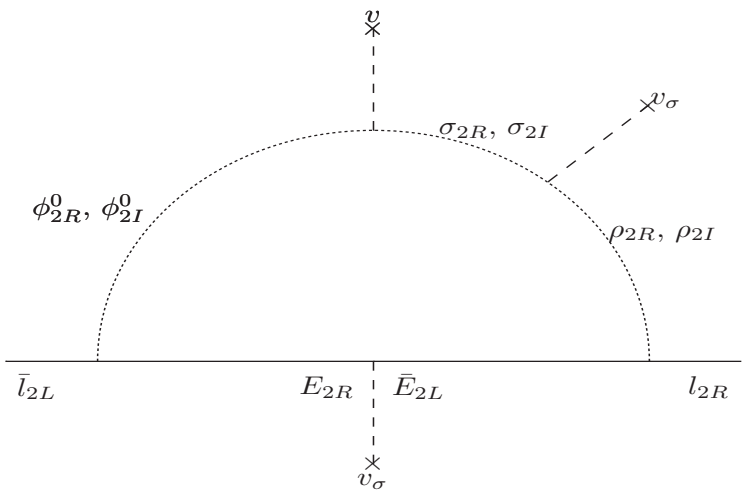
E_{3R}
$\bar{E}_{3L}$
l_{nR}
v_σ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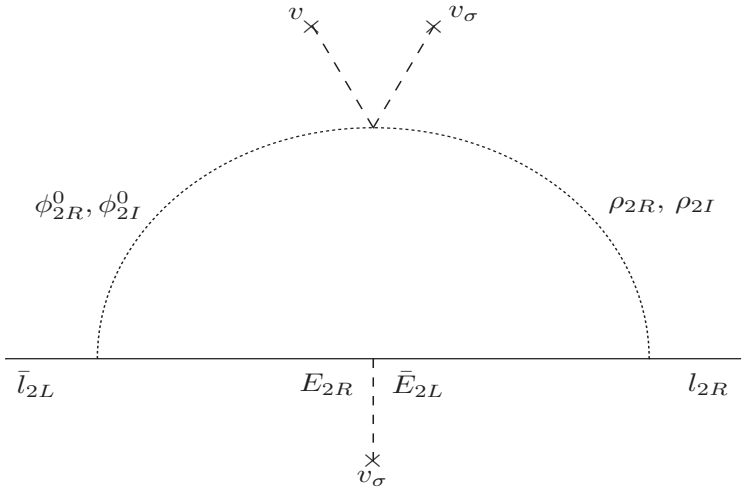
$\bar{l}_{2L}$
E_{2R}
$\bar{E}_{2L}$
l_{2R}
v_σ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v_σ
ϕ^0_{2R}, ϕ^0_{2I}
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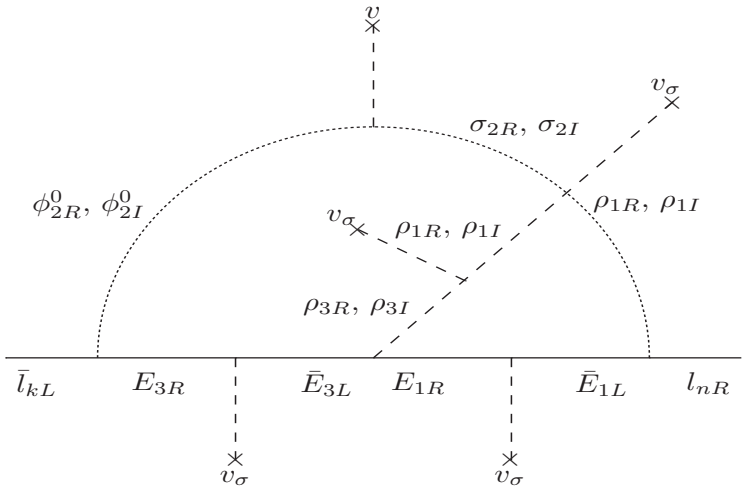
E_{2R}
$\bar{E}_{2L}$
l_{2R}
v_σ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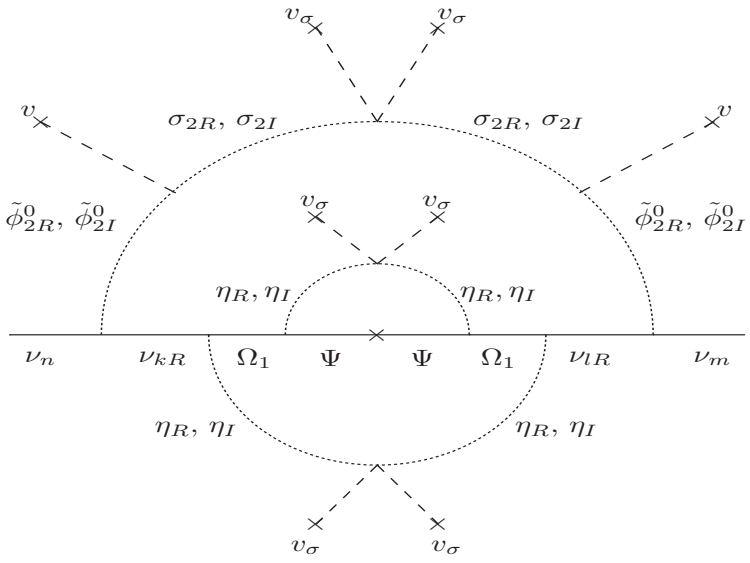
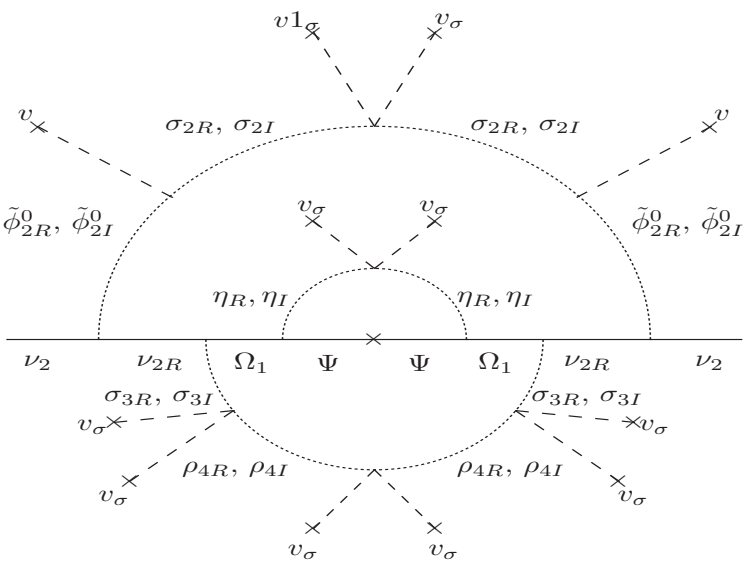
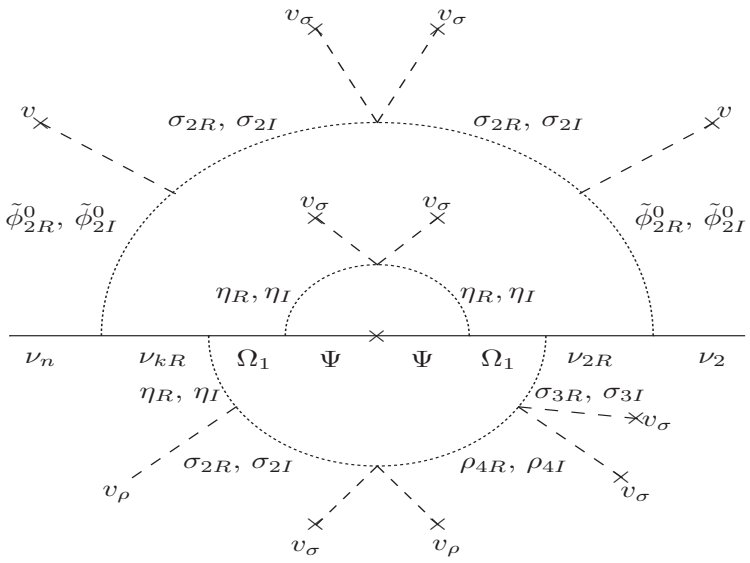
v_σ
$\rho_{1R},\ \rho_{1I}$
$\rho_{3R},\ \rho_{3I}$
$\bar{l}_{kL}$
E_{3R}
$\bar{E}_{3L}$
E_{1R}
$\bar{E}_{1L}$
l_{nR}
v_σ
v_σ


v_σ
v_σ
v
σ_{2R}, σ_{2I}
σ_{2R}, σ_{2I}
v
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_n
ν_{kR}
Ω_1
Ψ
Ψ
Ω_1
ν_{lR}
ν_m
η_R, η_I
η_R, η_I
v_σ
v_σ


$v1_\sigma$
v_σ
v
σ_{2R}, σ_{2I}
σ_{2R}, σ_{2I}
v
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_2
ν_{2R}
Ω_1
Ψ
Ψ
Ω_1
ν_{2R}
ν_2
σ_{3R}, σ_{3I}
σ_{3R}, σ_{3I}
v_σ
v_σ
ρ_{4R}, ρ_{4I}
ρ_{4R}, ρ_{4I}
v_σ
v_σ
v_σ
v_σ


v_σ
v_σ
v
σ_{2R}, σ_{2I}
σ_{2R}, σ_{2I}
v
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
v_σ
v_σ
$\tilde{\phi}^0_{2R}, \tilde{\phi}^0_{2I}$
η_R, η_I
η_R, η_I
ν_n
ν_{kR}
Ω_1
Ψ
Ψ
Ω_1
ν_{2R}
ν_2
η_R, η_I
σ_{3R}, σ_{3I}
v_σ
σ_{2R}, σ_{2I}
ρ_{4R}, ρ_{4I}
v_ρ
v_σ
v_σ
v_ρ

Conclusions

For the $S_3 \times Z_2$ flavor model:

- The SM fermion mass hierarchy is generated by the loops.
- The cutoff scale is $\Lambda \sim 2.5$ TeV.
- The model predicts one massless and two non-zero mass neutrinos.
- The mass scale of the non-SM particles are of the order of 1 TeV.
- The model possesses DM particle candidates.

For the IDM model with sequential loop suppression mechanism

- The first renormalizable model of sequential loop suppression mechanism without soft-breaking mass terms.
- Only the top quark and exotic fermions acquire tree-level masses.
- The masses for the bottom, strange and charm quarks, tau and muon leptons are generated at one-loop level, whereas the masses for the up and down quarks as well as the electron mass appear at two-loop level.
- Light active neutrino masses arise at three-loop level.
- The model has DM particle candidates.

Acknowledgements

Thank you very much to all of you for the attention.

A.E.C.H was supported by Fondecyt (Chile), Grant No. 1170803.

Extra Slides

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}}^{(D)} &= \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(d)} \overline{q}_{jL}\phi \left(B_R^{(k)}\eta\right)_{\mathbf{1}} \frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} x_{jk}^{(d)} \left(\overline{B}_L^{(j)}\eta\right)_{\mathbf{1}'} d_{kR} \frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{k=1}^{3} y_k^{(B)} \left(\overline{B}_L^{(k)} B_R^{(k)}\right)_{\mathbf{1}} \chi + h.c. \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}}^{(l)} &= \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(l)} \overline{l}_{jL}\phi \left(E_R^{(k)}\eta\right)_{\mathbf{1}} \frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} x_{jk}^{(l)} \left(\overline{E}_L^{(j)}\eta\right)_{\mathbf{1}'} l_{kR} \frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{k=1}^{3} y_k^{(E)} \left(\overline{E}_L^{(k)} E_R^{(k)}\right)_{\mathbf{1}} \chi \qquad (31)
\end{aligned}
$$

where I set

$$
v_\chi = \lambda\Lambda, \qquad \lambda = 0.225. \qquad (32)
$$

The charged fermion mass matrices are:

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)}\lambda^3 & \varepsilon_{12}^{(u)}\lambda^2 & y_{13}^{(u)}\lambda^2 \\ \varepsilon_{21}^{(u)}\lambda^2 & \varepsilon_{22}^{(u)}\lambda & y_{23}^{(u)}\lambda \\ \varepsilon_{31}^{(u)}\lambda & \varepsilon_{32}^{(u)} & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{33}$$

$$M_{D,l} = \begin{pmatrix} \varepsilon_{11}^{(d,l)}\lambda^4 & \varepsilon_{12}^{(d,l)}\lambda^3 & \varepsilon_{13}^{(d,l)}\lambda^2 \\ \varepsilon_{21}^{(d,l)}\lambda^3 & \varepsilon_{22}^{(d,l)}\lambda^2 & \varepsilon_{23}^{(d,l)}\lambda \\ \varepsilon_{31}^{(d,l)}\lambda^2 & \varepsilon_{32}^{(d,l)}\lambda & \varepsilon_{33}^{(d,l)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

where the dimensionless parameters $\varepsilon_{jk}^{(f)}$ $(j,k=1,2,3)$ with $f=u,d,l$, are generated at one loop level. The invariance of charged exotic fermion Yukawa interactions under the cyclic symmetry requires to consider the Z_8 instead of the Z_4 discrete symmetry.

$$
\begin{aligned}
\mathcal{L}_Y \;=\;& y_{3j}^{(u)}\overline{q}_{3L}\widetilde{\phi}_1 u_{3R} + \sum_{n=1}^{2} x_n^{(u)}\overline{q}_{nL}\widetilde{\phi}_2 T_R + \sum_{n=1}^{2} w_j^{(u)}\overline{T}_L\sigma_2 u_{nR} + \sum_{n=1}^{2} z_n^{(u)}\overline{\widetilde{T}}_{1L}\rho_1 u_{nR} + y_T\overline{T}_L\sigma_1 T_R \\
&+ \sum_{n=1}^{2} m_{\widetilde{T}_n}\overline{\widetilde{T}}_{nL}\widetilde{T}_{nR} + \sum_{n=1}^{2} x_n^{(T)}\overline{T}_L\rho_n\widetilde{T}_{nR} + \sum_{n=1}^{2} x_n^{(d)}\overline{q}_{3L}\phi_2 B_{nR} + \sum_{n=1}^{2}\sum_{j=1}^{3} y_{nj}^{(d)}\overline{B}_{nL}\sigma_2^* d_{jR} \\
&+ \sum_{n=1}^{2}\sum_{m=1}^{2} y_{nm}^{(B)}\overline{B}_{nL}\sigma_1^* B_{mR} + \sum_{j=1}^{3} w_j^{(d)}\overline{\widetilde{T}}_{2L}\varphi_1^+ d_{jR} + \sum_{j=1}^{3} z_j^{(d)}\overline{T}_L\varphi_2^+ d_{jR} \\
&+ \sum_{k=1,3} x_{k3}^{(l)}\overline{l}_{kL}\phi_2 E_{3R} + \sum_{k=1,3} y_{3k}^{(l)}\overline{E}_{3L}\rho_2 l_{kR} + x_{22}^{(l)}\overline{l}_{2L}\phi_2 E_{2R} + y_{22}^{(l)}\overline{E}_{2L}\rho_2 l_{2R} \\
&+ z_{31}^{(E)}\overline{E}_{3L}\rho_3 E_{1R} + \sum_{k=1,3} y_{1k}^{(l)}\overline{E}_{1L}\rho_1 l_{kR} + \sum_{i=1}^{3} y_i^{(E)}\overline{E}_{iL}\sigma_1^* E_{iR} + x_2^{(\nu)}\overline{l}_{2L}\widetilde{\phi}_2\nu_{2R} \\
&+ \sum_{k=1,3}\sum_{n=1,3} x_{kn}^{(\nu)}\overline{l}_{kL}\widetilde{\phi}_2\nu_{nR} + \sum_{k=1,3} y_k^{(\Omega)}\overline{\Omega_{1R}^C}\eta^*\nu_{kR} + y^{(\Omega)}\overline{\Omega_{1R}^C}\sigma_3^*\nu_{2R} \\
&+ x_1^{(\Psi)}\overline{\Omega_{1R}^C}\eta\Psi_R + x_2^{(\Psi)}\overline{\Omega_{2R}^C}\eta^*\Psi_R + z_\Omega\overline{\Omega_{1R}^C}\sigma_2^*\Omega_{2R} + m_\Psi\overline{\Psi_R^C}\Psi_R + h.c \qquad (1)
\end{aligned}
$$